HW 7

ATS 541

Probs. 1-4

1. 10 pts
2. 10
3. 10
4. 20

50 pts

1. Radiation fog problem

10 pts

Given:

$$\Delta q_{rad} = -1 \text{ cal g}^{-1} = 4.186 \times 10^3 \text{ J kg}^{-1}$$

$T_{sp} = 10\ °C$ $\qquad e_s(10°C) = 12 \text{ mb} = 1200 \text{ Pa}$

$p = 100 \text{ kPa}$

a) From eq. (~~##~~ 7.3) of notes (~~Fall 1994~~ 10/20/98 version)

$$dq = \left( c_p + \frac{\varepsilon L_{vl}^2 e_s}{p R_v T^2} \right) dT$$

Assuming that the quantity in parentheses is constant during integration (i.e., $\Delta T$ is small) we can write

$$\Delta T \doteq \frac{-\Delta q}{c_p + \frac{\varepsilon L_{vl}^2 e_s}{p R_v T^2}}$$

$$= \frac{-4.186 \times 10^3 \text{ J kg}^{-1}}{1005 \text{ J kg}^{-1}\text{K}^{-1} + \frac{0.622 (2.5 \times 10^6 \text{ J kg}^{-1})^2 (1200 \text{ Pa})}{(100{,}000 \text{ Pa})(461 \text{ J kg}^{-1}\text{K}^{-1})(283.15)^2}}$$

$$= -1.84 \text{ K}$$

$$T_f = T_{sp} + \Delta T = 10 - 1.84 = \boxed{8.16\ °C}$$

b) $\Delta e_s$ can be obtained from (~~#23~~ ~~7.2~~ 7.4) of Notes

$$\Delta e_s = \frac{\Delta q}{\frac{c_p R_v T^2}{L_{vl} e_s} + \frac{\varepsilon L_{vl}}{p}}$$

(same assumption as in (a))

Plugging in gives a value of $\boxed{\Delta e_s = -1.5 \text{ mb.}}$

c) $\cancel{\Delta C}$ $\Delta M$ is obtained from (~~#25~~)(7.5)

$$\cancel{\Delta C}\ \Delta M \doteq \frac{-\Delta e_s}{R_v T} = 1.14\ g\ m^{-3}$$

$$= \frac{\cancel{(-150\ kg\ m^{-1}\ s^{-2})}}{(461\ m^2 s^{-2} K^{-1})(233\ K)} = 1.15 \times 10^{-3}\ kg\ m^{-3} = \boxed{1.15\ g\ m^{-3}}$$

10 pts

2. The average precip. rate inside the ITCZ is 10 mm $day^{-1}$.

a) Calc. the average column heating in $W\ m^{-2}$. Use $\rho_W = 10^3\ kg\ m^{-3}$

The latent heating rate $\dot{Q}_L$ is $L \cdot R_{ppn}$, where $R_{ppn}$ is the precip. rate in units of $kg\ s^{-1}\ m^{-3}$

Rate of precipitation:

$$R_{ppn} = \frac{10\ mm}{day} \cdot \frac{1\ day}{86{,}400\ s} \cdot \frac{10^3\ kg}{m^3} \cdot \frac{1\ m}{10^3\ mm}$$

$$= 1.157 \times 10^{-4}\ kg\ s^{-1}\ m^{-2}$$

Then $\dot{Q}_L = L \cdot R_{ppn} = (2.5 \times 10^6\ J\ \cancel{kg^{-1}})(1.157 \times 10^{-4}\ \cancel{kg}\ s^{-1}$

$$= 289\ J\ s^{-1}\ m^{-2}$$
$$= 289\ W\ m^{-2}$$

b) Compare with longwave flux

$$F = \sigma T^4$$
$$= (5.67 \times 10^{-8}\ W\ m^{-2} \cdot K^{-4})\ (300\ K)^4$$
$$= 459\ W\ m^{-2}$$

$$\therefore \frac{\dot{Q}_L}{F} = 0.63$$

10 pts. 3 Moist air moves inland from a maritime region and cools via contact (negative heat flux) with the ground Calculate the T at which fog forms if the air is initially $T = 16°C$, $f = 0.66$

Calculate the dew point (or the saturation pt.)

$$f \doteq \frac{e}{e_s} = \frac{e_s(T_d)}{e_s(T)}$$

$$e_s(T_d) = e_s(T) \cdot f = e_s(16°c) \cdot 0.66$$
$$= 18.17 \text{ mb} \cdot 0.66 = 12.00 \text{ mb}$$

Then $T_d$ corresponds to an $e = 12.00$

$\Rightarrow T_d = 9.65\ °c$

20 pts **4.** $T = -10°C$

15

$f_0 = 0.50$

a) Find $f$ for this air when $T$ is increased to $22°C$

6 pts

$$e_1 = e_s(-10°C) * 0.50$$
$$= 2.86 \text{ mb} * 0.50$$
$$= 1.43 \text{ mb}$$

$$f_1 = \frac{e}{e_s(22°C)} = \frac{1.43 \text{ mb}}{26.43 \text{ mb}} = \boxed{5.4\ \%}$$

b) Find mass of water need to raise the $f$ to 40%. Assume: $T = \text{const} = 22°C$

6 pts

$$f_2 = \frac{e}{e_s(22°C)}$$

$$e_2 = f_2\, e_s(22°C) = (0.40)(26.43 \text{ mb})$$
$$= 10.57 \text{ mb}$$

$1 \text{ mb} = 10^2 \text{ Pa}$

Then $e_2 = \rho_v R_v T \Rightarrow \rho_{v2} = \frac{e_2}{R_v T} = \frac{1057 \text{ Pa}}{(461 \text{ J kg}^{-1}\text{K}^{-1})(295.1}$

$$= 7.769 \times 10^{-3} \text{ kg m}^{-3}$$

$M_{v2}$ = Total mass $= \rho_{v2} * V = (1.942 \times 10^{-2}) * (75 \text{ m}^3)$

$$\boxed{= 0.583 \text{ kg}}$$

8. (cont.)

Original mass of water:

$$\rho_{v1} = \frac{e_1}{R_v T} = \frac{143}{461 \cdot 295.15} = 1.051 \times 10^{-3} \ kg\ m^{-3}$$

$$M_{v1} = \rho_{v1} \cdot V = 0.079 \ kg.$$

$$\Delta M = M_{v2} - M_{v1} = (0.583 - 0.079)\ kg = \boxed{0.504 \ kg.}$$

c) Energy required to maintain the (isothermal) condition. (Evap. the water and maintain a final $T = 22°C$)

8 pts

Evaporation: $\Delta Q = L_{ve} \cdot M = (2.45 \times 10^6 \ J\ kg^{-1})(.504\ kg)$

$$= 1.23 \times 10^6 \ J$$

Also need to consider the sensible heating of dry and moist air:

$$c_p \Delta T m_d + c_{pw} \Delta T m_w = (c_p m_d + c_{pw} m_w)\Delta T$$

$$= [(1005 \ J\ kg\ K^{-1} \cdot 99.2\ kg) + 1461 \cdot .504\ kg)] \cdot 32\ K$$

$$= 3.21 \times 10^6 \ J$$

Assume $p = 1000$ mb

$p_d = p - e = 998.6$ mb

$$\text{Total} = (3.21 + 1.23) = \boxed{4.44 \times 10^6 \ J}$$

$$\rho_d = \frac{p_d}{R_d T} = \frac{m_d}{V}$$

$$m_d = \frac{p_d V}{R_d T}$$

$$= \frac{(p - e) V}{R_d T} = 99.2 \ kg$$