Growth of raindrops

Do raindrops looks like "tear" drops?

http://www.ems.psu.edu/~fraser/Bad/BadRain.html http://www.fluidmech.net/tutorials/raindrops/raindrop.htm

Initiation of rain in warm clouds

- Fact: The observed time scale for growth of precipitation to raindrop size (d ~ 1 mm) is about 20 min.
- A significant amount of rain in the tropics originates from shallow (warm) clouds (T > 0 °C).
- Once raindrops attain a radius (r) of 20 μm, growth by collision-coalescence becomes more probable (observations and theory).
- For drops with r > 30 μm, coalescence is the dominant process of drop growth (to be demonstrated).

What mechanism(s) produce drops > 20 μ m?

- Mixing between cloud and environment:
 - Homogeneous mixing: mixing is complete, leads to a homogeneous mixed volume of similar e/e_s
 - Inhomogeneous mixing: evaporation is local and leads to volumes of air with no or very few drops; then subsequent mixing occurs and reduces N_d; these drops can grow via larger (S-1)
 - Recall the sources and sinks of S from the dS/dt equation
 - Cloud top vs lateral mixing
- Mixing (evaporation) has the largest impact on the smallest drops. Why?
 - The net effect is to reduce the drop concentration. How would this impact subsequent growth?

We are considering this in Chap. 8:

Water vapor Nucleation Growth by diffusion Cloud droplets Collection Rain drops



FIG. 8.10. A simplified schematic diagram of possible interactions between various categories of cloud particles. The bold arrows show the dominant pathways by which water vapor is transformed into rain via the "warm-rain" process (left-hand side) and via the "cold-rain" process (right-hand side) in heavy rain situations. The dotted set of arrows near the center of the diagram identifies a likely cyclical process for generating secondary ice particles via the rime-splintering mechanism of Hallett and Mossop (1974). The form of the diagram was adapted from Rutledge and Hobbs (1984).

Collection

 Growth process resulting from drop (or ice particle) collisions and subsequent coalescence

Ultra giant CCN

- Very large CCN are more likely in maritime regions.
- Does this influence rate of growth by condensation?

Sabine Göke, S., H.T. Ochs III, and R.M. Rauber, 2007: Radar Analysis of Precipitation Initiation in Maritime versus Continental Clouds near the Florida Coast: Inferences Concerning the Role of CCN and Giant Nuclei. Journal of the Atmospheric Sciences, Volume 64, Issue 10 (October 2007) pp. 3695–370.

Basic conclusion:

The characteristic radar reflectivity threshold for precipitation development was reached at a much lower altitude above cloud base in a much faster time in the truly maritime clouds. This result supports the conclusions of Hudson and Yum that precipitation development in the SCMS clouds was primarily controlled by CCN concentrations rather than giant nuclei (GN) concentrations.

The continuous collection model

Important aspects of collection:

- a) Drop terminal fall speeds (variation)
- b) Collision efficiency
- c) Growth equations (models: simple to complex)
 - Continuous collection model (Bowen model)
 - Statistical growth (Telford model)
 - Statistical growth (stochastic collection model)
 - Condensation + stochastic collection

A grazing trajectory for the collected drop (r_s) as the flow field around the collector drop (r_1) tends to move the smaller drop aside. Only those drops whose centers lie within the distance R of the fall axis of the collector drop are assumed to be collected. Fig. 7.1 from Young (1993)



Flow regimes about a sphere, as a function of Reynolds number* Fig. 7.2 from Young (1993)



Ratio of inertial forces (ρV^2) to viscous forces ($\mu V/L$). The variables are V_T and r (and V_T is a function or r)

Computed shapes of freely falling water drops at terminal velocity in air for diameters shown. Taken from Fig. 7.3 of Young (1993) who adapted it from Beard and Chuang (1987). Note: drops are spherical for diameters \leq 1 mm.



Oblate spheroid shape for large drops

http://en.wikipedia.org/wiki/Wikipedia:Featured_picture_candidates/Raindrop_shapes

Terminal fall speeds (pp. 124-125)

- Force balance: gravity vs.
 frictional drag
- Drag: $F_{drag} = (\pi/2)r^2V_T^2\rho C_D$ $F_{drag} = 6\pi\mu rV_T(C_DRe/24) - (Re = 2\rho V_T r/\mu)$
- Gravitational force: $F_g = mg = (4/3)\pi r^3(\rho_L \rho_{air})g$, or since $\rho_L >> \rho_{air}$, $F_g = (4/3)\pi r^3 g \rho_L$



Equivalent Drop Radius (um)

Terminal fall speeds of drops

a) Stokes Law regime (r < 30 μ m)

$$V_T = \frac{2}{9} \frac{r^r g \rho_L}{\mu} = k_1 r^2$$
 (8.5)

b) Intermediate drop sizes

(40 μm < r < 0.6 mm)

$$V_{T} = k_{3}r, k_{3} = 8x10^{3} s^{-1}$$
 (8.8)

c) Large drop sizes (0.6 mm < r < 2 mm) $V_T = k_2 r^{1/2}$, $k_2 = 2.2 x 10^3 (\rho_0 / \rho)^{1/2} \text{ cm}^{1/2} \text{ s}^{-1}$ (8.6)

Atlas (1973) relation for raindrops: $V_T(D) = 3.78 D^{0.67}$ D < 3 mm $V_T(D) = 9.65 - 10.3e^{-0.6D}$ D > 3 mm $(V_T \text{ in m/s, D in mm})$



Diam. (mm)	Fall speed (m/s)	Diam. (mm)	Fall speed (m/s)		
0.1	0.27 drizzle	2.6	7.57		
0.2	0.72	Large 2.8	7.82		
0.3	1.17	raindron 3.0	8.06		
0.4	1.62	3.2	8.26		
0.5	2.06	3.4	8.44		
0.6	2.47	3.6	8.60		
0.7	2.87	3.8	8.72		
0.8	3.27	4.0	8.83		
0.9	3.67 Small	4.2	8.92		
1.0	4.03 Small	4.4	8.98		
1.2	4.64	4.6	9.03		
1.4	5.17	4.8	9.07		
1.6	5.65	5.0	9.09		
1.8	6.09	5.2	9.12		
2.0	6.49	5.4	9.14		
2.2	6.90	5.6	9.16		
2.4	7.27	5.8	9.17		

TABLE 8.1. Terminal Fall Speed as a Function of Drop Size (equiva-
lent spherical diameter) (From Gunn and Kinzer, 1949)

Sea level: p = 1013.25 hPa, T = 20 °C

asymptote

Collision efficiency

Definition:

Fraction of droplets with radius r (r_s) in the path (defined by airflow around the falling drop and inertial effects of the small drop) swept out by the collector drop of radius R.

 $E(R,r) = x_0^2 / (R + r)^2$



 $E(R,r) = x_0^2 / (R + r)^2$

Low for small values of r/R (small r)

Drops with small r have very small inertia and follow airflow.

Relative maxima near r/R = 0.6 for intermediate sizes of R > 20 μ m. Also, large gradient in E near r=20 μ m.

This is explains why coalescence is not effective until droplet sizes attain r=20 μm.

Effect at large $r/R \sim 1$: wake capture.



TABLE 8.2. Collision Efficiency E for Drops of Radius R Colliding with Droplets of Radius r. (Data for $R < 50 \mu m$ adapted from Klett and Davis, 1973; for 50 $\mu m \leq R \leq 500 \mu m$ from Beard and Ochs, 1984; for $R > 500 \mu m$ from Mason, 1971)

	$r(\mu m)$										
$R(\mu m)$	2	3	4	6	8	10	15	20	25		
10	0.017	0.027	0.037	0.052	0.052						
20	*	0.016	0.027	0.060	0.12	0.17	0.17				
30	*	*	0.020	0.13	0.28	0.37	0.54	0.55	0.47		
40	*	*	0.020	0.23	0.40	0.55	0.70	0.75	0.75		
50		-	0.030	0.30	0.40	0.58	0.73	0.75	0.79		
60		0.010	0.13	0.38	0.57	0.68	0.80	0.86	0.91		
80		0.085	0.23	0.52	0.68	0.76	0.86	0.92	0.95		
100		0.14	0.32	0.60	0.73	0.81	0.90	0.94	0.96		
150	0.025	0.25	0.43	0.66	0.78	0.83	0.92	0.95	0.96		
200	0.039	0.30	0.46	0.69	0.81	0.87	0.93	0.95	0.96		
300	0.095	0.33	0.51	0.72	0.82	0.87	0.93	0.96	0.97		
400	0.098	0.36	0.51	0.73	0.83	0.88	0.93	0.96	0.97		
500	0.10	0.36	0.52	0.74	0.83	0.88	0.93	0.96	0.97		
600	0.17	0.40	0.54	0.72	0.83	0.88	0.94	0.98	+		
1000	0.15	0.37	0.52	0.74	0.82	0.88	0.94	0.98	+		
1400	0.11	0.34	0.49	0.71	0.83	0.88	0.94	0.95	t		
1800	0.08	0.29	0.45	0.68	0.80	0.86	0.96	0.94	ŧ		
2400	0.04	0.22	0.39	0.62	0.75	0.83	0.92	0.96	+		
3000	0.02	0.16	0.33	0.55	0.71	0.81	0.90	0.94	†		

—Collision efficiency less than 0.01.

* Value cannot be determined accurately from available data.

†Value close to one.

Collision efficiency as a function of R and r, based on the data in Table 8.2. Contours are values of the E.

Fig. 8.3 from R&Y

Small droplets (r) need to be larger the several μ m in order for E to be sufficiently large.

The optimum large drop (R) appears to be about 700 μ m, or 0.7 mm

FIG. 7.5. Coalescence efficiencies as a function of the Weber number, as determined experimentally by Ochs et al. (1986). (**m**), Present data; (—), Beard and Ochs (1983). *Journal of Atmospheric Sciences*, **43**, 3. American Meteorological Society. Reproduced with permission.

Ratio of dynamic pressure to pressure across a curved interface (σ_{LV} is surface tension)

 $N_{we} = \frac{\rho_L r_L}{\sigma_{LV}} (W_L - W_s)^2$

FIG. 7.6. The field of collection efficiencies (per cent) based on experiment and theory, as discussed in the text. Isopleths in the cross-hatched regions are based on experimental values from (clockwise from lower left) Woods and Mason (1964), Beard and Pruppacher (1971), Beard and Ochs (1983), and Woods and Mason (1965).

Fig. 7.6 from Young (1993)

Growth Equations

$$\frac{dR}{dt} = \frac{\overline{E}M}{4\rho_L} u(R)$$
(8.15)

$$\frac{dR}{dz} = \frac{dR}{dt}\frac{dt}{dz} = \frac{dR}{dt}\frac{1}{U-u(R)}$$

$$\frac{dR}{dz} = -\frac{\overline{E}M}{4\rho_L} \frac{u(R)}{U - u(R)} \approx -\frac{\overline{E}M}{4\rho_L}$$
Important parameters: (8.16)

Important parameters: M, U

The above approximation is valid only when the updraft U is *small* compared to the terminal fall speed u(R)

Results from the Bowen model (simple)

Fig. 8.5 from R&Y. Drop trajectories for the collision efficiencies of Table 8.2 and Fig. 8.3, assuming a coalescence efficiency of 1.0. Initial drop radius is 20 μ m, cloud water content is 1 g m⁻³, and cloud droplet radii are 10 μ m.

The time scale is too large by about a factor of 2.

IG. 8.6. Drop diameters for the trajectories of Fig. 8

Statistical growth: the Telford model

The statistical-discrete capture process is crucial in the early stages of rain formation. Chances captures during the early stages of the collection process are important. Reasonable drop spectra evolved over periods of a few 10's of minutes.

Based on some assumed distribution of probability of collision.

$$p(m) = e^{-\overline{n}V} \frac{(\overline{n}V)^m}{m!}$$

n – avg drop conc.

m – number of droplets in a volume V

Collected droplets of uniform size of 10 µm.

Collector drops have twice the volume, r = 12.6 μm

FIG. 8.7. Schematic illustration of droplet growth by discrete captures, including statistical variability. Shown are four realizations of the growth of a drop by collection of smaller, equal-sized droplets. The average growth for many such realizations is indicated by the dashed line.

Some early work on drop growth by stochastic collection Some results

FIG. 8.10. Example of the development of a droplet spectrum by stochastic coalescence. (From Berry and Reinhardt, 1974b.)

Initial unimodel distribution evolves to a bimodel distribution (cloud & rain drops)

Three basic modes of collection operate simultaneously to produce large drops:

- b) Autoconversion adds water to S2 so that the other modes can operate.
- c) Accretion is the main mechanism for transferring water from S1 to S2.
- d) Large hydrometeor self capture

produces large drops quickly and is responsible for the rapid increase in r_g and the emerging shape of S2.

Initial spectrum S1: r = 10 μ m, M = 0.8 g m⁻³ Initial spectrum S2: r = 20 μ m, M = 0.2 "

Condensation plus stochastic coalescence. Condensation maintains the supply of water droplets for the growing raindrops, and is important.

FIG. 8.12. The effect of condensation on growth by coalescence. The same droplet spectrum evolves by coalescence, (a) without, and (b) with, an allowance for condensation. (Adapted from Ryan, 1974.)

Maritime cloud, with $N_c = 105S^{0.63}$. The collection process removes many of the cloud droplets after 500 s (N decreases), and therefore S increases. Caclulations with condensation + coalescence

FIG. 8.13. Time histories of supersaturation (%) and droplet concentration (cm⁻³) in a cloud formed on a maritime distribution of condensation nuclei. (Adapted from Ochs, 1978.)

Same is Fig. 8.13, except for a continental cloud, with $N_c = 1450S^{0.84}$. In this case the coalescence process is not effective, and the dramatic rise in S does not occur.

FIG. 8.14. As Fig. 8.13, except for a continental distribution of condensation nuclei. (Adapted from Ochs, 1978.)

Quasi-stochastic model Figure taken from Young (1993)

This again illustrates that stochastic processes are important in the early stages of raindrop growth

FIG. 7.7. The growth of 10 μ m radius drops collecting 8 μ m radius drops for the continuous, discrete, and Poisson collection models. The expected number of collection events within a given time step (Δ t) is 0.5. Numbers within the circles reflect the percentage of drops of that size; arrows show growth paths. From Young (1975). *Journal of Atmospheric Sciences*, **32**, 5. American Meteorological Society. Reproduced with permission.

Drop size distributions (Chap. 10): What controls the shape of the distribution?

More on the sources/sinks

- Source: autoconversion represents the appearance of drops large enough to be effect collectors (previous growth by diffusion, Chap. 7)
- Growth: by collection (coalescence) (Chap. 8)
- Breakup: due to hydrodynamic instability of large drops, and collisions among drops

The slope of the distribution changes (or may change) as the rainfall rate changes

Fig. 10.2, R&Y

Modes of oscillations of raindrops applies to large raindrops

Drop breakup

- Aerodynamic instability due to flow around fast-falling drop (surface tension is relatively small for large drops)
 - Begins to be important when D = 3 mm
 - Drops with D > 6 mm are unstable and have short lifetimes
- Collisions among drops
 - Coalescence efficiency is a factor: high for R<0.4 mm and r<0.2 mm

Fig. 10.3. Coalescence efficiency as a function of drop radii, r and R. The values represent the fraction of collisions that result in coalescence.

Fig. 10.4. Theoretical DSD produced after 30 min in a model of droplet growth that includes the effects of: Condenstation Coalescence Collision breakup

Four observed modes of drop breakup: (a) filament, (b) sheet, (c) disk, and (d) bag. Taken from Young (1993).

The field of drop breakup probabilities based theory (consideration of collision kinetic energy). Dots represent where observed values exist. Taken from Young (1993).

Measurements of raindrops

- Aircraft probes
- Ground-based instrumentation
 - Disdrometers
 - Momentum impact
 - optical
 - Radar
 - Doppler
 - Dual polarization

Laser probes measure very large raindrop in Hawaii clouds 2-3 km deep. Courtesy K. Beard.

Disdrometer

Measures the momentum impact (energy) of raindrops. The impact energy is proportional to raindrop radius, assuming that the raindrops are falling at terminal speed.

$$M = mV_T = \frac{4}{3}\pi r^3 \rho_L V_T(r) = f(r^n)$$

OTT Parsivel disdrometer

Description

• laser-based optical system for the measurement of precipitation characteristics (see 3rd bullet)

• precipitation particles are differentiated and classified as drizzle, rain, sleet, hail, snow or mixed precipitation

•Measurements: size and the vertical velocity of each individual precipitation particle, from which the size spectrum, precipitation rate, the equivalent radar reflectivity factor, the visibility and the kinetic precipitation energy as well as the type of precipitation (2nd bullet) are derived.

http://www.ott-hydrometry.de/web/ott_de.nsf/id/pa_parsivel_e.html#

More details

http://vortex.nsstc.uah.edu/mips/data/current/surface/index.html

Disdrometer measurements

Fig. 7. Observed and fitted composite spectra for the profiler's (a) shallow convective, (b) deep convective, (c) mixed stratiform–convective, and (d) stratiform rain from 6, 21, 115, and 106 spectra, respectively, for rainfall rates of 5 mm h⁻¹. (Tokay et al. 1997)

Time dependent drop size distribution (bow echo)

- 1. Drop size was largest during the initial convection accompanying the bow passage
- 2. Other maximum values were obtained during convective cell passage (915 Reflectivity)
- 3. As the precipitation ended drop size decreased

Fig. 13. NCAR CP2 X-band radar reflectivity evolution of two small cumulus clouds on 5 and 10 Aug 1995. Reflectivity calculated from SCMS composite droplet distributions are shown for their corresponding 0.5-km layers on (B), (C), (F), and (G). Radar scan times (UTC) and azimuth angle are shown for each panel. From Laird et al 2000.

Evolution of radar echo in shallow Hawaiin cumulus clouds. Vertical sections of radar reflectivity factor showing the evolution from just after "first echo."

These cloud systems develop large drops and Z values.

Fig. 6. (f) Vertical section along the Y"=0 cut plane. Outer solid line is the 10-dBZ contour of reflectivity. Gray scales depict Z_{DR} starting at 0 dB and incrementing 1 dB. Arrows are cell-relative wind vectors in the Y"=0 cut plane. (g) Contours of Z_{DR} start at 0 dB andincrement by 1 dB (note a vertical column of numbers on upper right side of figure next to the gray scale bar). Gray scale depicts LDR starting at -24 dB (outer dashed line) and increments by 3 dB. (h) As in (g) except gray scales depict A3 starting at 0.5 dB km⁻¹ (outer dashed line) and increments by 0.5 dB km⁻¹.

Fig. 4a. Aircraft measurements of thermodynamic and microphysical parameters during cloud passes 1–3 on 23 July 1985. The top panel for each pass illustrates the size of each drop measured by the 2D-P probe during the pass as a function of time (distance along the flight path). The bottom panel shows the updraft velocity and JW measurement of cloud liquid water. FSSP cloud droplet spectra are shown to the right of the panels, and 2D-P raindrop spectra are shown on the bottom of the figure. These spectra were averaged over regions indicated by arrows on the top pass of each of the panels. Calculated reflectivities and rainfall rates are shown for each of the raindrop spectra. From Szumowski et al 1998

Fig. 5. Raindrop axis ratios as a function of diameter. Shown are mean axis ratios (symbols) and standard deviations (vertical lines) for the aircraft observations of Chandrasekar et al. (diamonds), the laboratory measurements of Beard et al. (triangles), Kubesh and Beard (squares), and present experiment (circles). Curves are shown for the numerical equilibrium axis ratio (αN) from <u>Beard and Chuang</u> (1987). the radar-disdrometer-derived axis ratios of Goddard and Cherry (1984), the empirical formula (αW) from the wind tunnel data of Pruppacher and Beard (1970), and the present fit to axis ratio measurements (αA). The shaded region covers the range for previous estimates of the equilibrium axis ratio (see <u>Table 3</u>). From Andsager et al 1999.

Electrostatic effects (extra)

Radar

ARMOR - Advanced Radar for Meteorological and Operational Research

C-band (5.5 GHz frequency, 5.5 cm wavelength)

dual polarization

MAX - Mobile Alabama X-band

X-band (9.4 GHz frequency, 3.3 cm wavelength) dual polarization

Radar equation
$$P_{r} = \frac{\pi^{3}c}{1024 \ln 2} \left[\frac{P_{t}\tau G^{s}\theta^{2}}{\lambda^{2}} \right] K^{2} \left[\frac{Z}{r^{2}} \right]_{\infty}$$

Radar reflectivity factor $Z = \int_{0}^{\infty} N(I)$

$$Z = \int_{0}^{\infty} N(D)D^{6}dD = \sum_{volume} n_{i}D_{i}^{6}$$

Use of radar to infer rainfall rate

horizontal polarization only

dual polarization

Vertically-pointing radar measurements

Homework

Chapter 8:

8.1, 8.2, 8.4

Chapter 10:

10.3 (optional extra credit for 441 students)