Chap. 3

ATS 541 HW 3

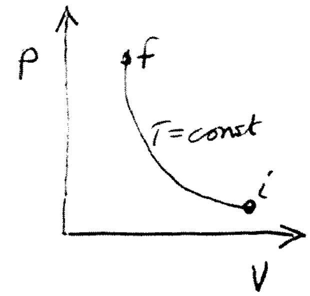
Solutions

~~100~~ ~~65~~ pts. total

	1.	20	(grads only)
	2.	12	
	3.	10	
	4.	10	
	5.	5	
	6.	10	
	7	~~8~~	10
5.4	~~4.2~~	~~10~~	20
5.7	~~4.5~~	~~10~~	10
5.10	~~4.7~~	~~15~~	10
		~~83~~	100

Total: 97 for undergrads
117 for grads

1. (a) isothermal reversible compression



pts. 20

$$W = \int_{V_i}^{V_f} p\, dV \qquad pV = nRT$$

$$V = \frac{nRT}{p}$$

$$dV = -nRT \frac{dp}{p^2} \left[+ \frac{nR}{p} \cancel{dT}^{\,0} \right]$$

$$W = \int_{p_i}^{p_f} -nRT \frac{dp}{p}$$

$$\boxed{W = -nRT \ln \frac{p_f}{p_i}} \quad \text{or} \quad W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$\Delta u = \int du = \int_{T_i}^{T_f} c_v dT = 0 \quad \text{since } T = \text{const.}$$

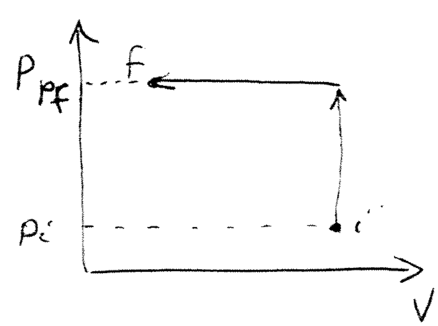
— — —

b) sudden compression to p_f (from p_i)

An example like this was given in the notes.

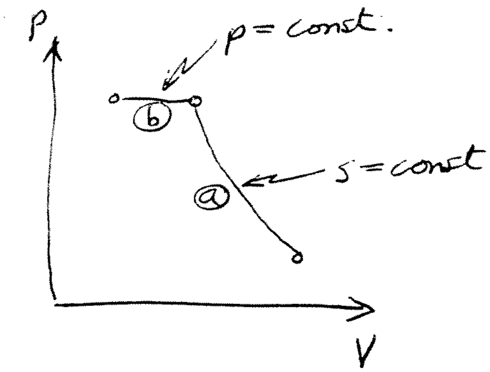
$$W = \int_{V_i}^{V_f} p\, dV = \underbrace{p_{ext} (V_f - V_i) = p_f}_{\text{same}} (V_f - V_i)$$

$$\boxed{W = p_f (V_f - V_i)}$$



Again, $\Delta u = 0$ since $T_f = T_i$

c) adiabatic reversible compression followed by a reversible isobaric cooling.



Need to consider 2 processes a and b.

Process a is adiabatic

$$dq = 0 \rightarrow dw_a = -du$$

Process b is const. pressure, so $dw_b = p\,dV$

$$\therefore \quad W = W_a + W_b$$

$$= -(u' - u_i) + p_f(V_f - V')$$

(primed values are at intermediate point)

$$= -nc_v(T' - T_i) + p_f(V_f - V')$$

$$= -nc_v(T' - T_i) + nR(T_f - T') \quad \leftarrow pV = nRT$$

$$= (nc_vT + nRT) - (nc_vT' - nRT') \qquad \text{note } p' = p_f$$

$$= nc_p(T - T') \qquad [c_p = \quad c_p = c_v + R]$$

$$= -nc_pT\left(\frac{T'}{T} - 1\right)$$

$$\frac{T'}{T_i} = \left(\frac{p'}{p_i}\right)^{R/c_p} \quad (p_f = p')$$

$$\boxed{W = -nc_pT\left[\left(\frac{p_f}{p_i}\right)^{R/c_p} - 1\right]}$$

$\boxed{\Delta u = 0}$ since $T_f = T_i$

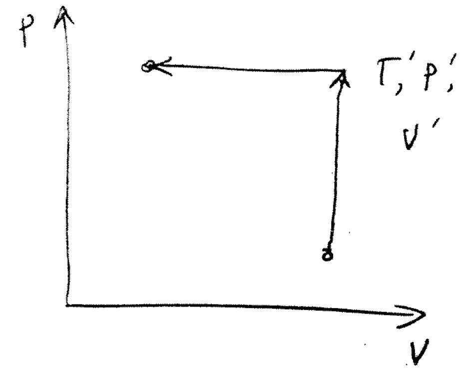
d) ① rev. increase of T @ const V [followed by] ③

② rev. decrease of T @ const p

Process ① :

$W_1 = 0$ since $V' = V_1$

$\Delta U = n c_v (T' - T_i)$



Process ②:

sign correction

$$W_2 = \int_{V'}^{V_f} p\,dV = -p_f(V_i - V_f) = -p_f\left[\frac{nRT_i}{p_i} - \frac{nRT_f}{p_f}\right]$$

$T_i = T_f = T$

$$W_2 = nRT\left(\frac{p_f}{p_i} - 1\right)$$

$$\Delta U = n C_v (T' - T_f)$$

$$\boxed{W = W_1 + W_2 = nRT\left(\frac{p_f}{p_i} - 1\right)}$$

$\Delta U = 0$

Grading:
1 pts: $\Delta U = 0$ in all cases

4 pts. ea: W in each part.

2. 12 pts

$p_1 = 1000$ mb $\quad T_1 = 10°C \quad$ dry air

$p_2 = 700$ mb

This is an adiabatic process, so $dq = 0$.

3 pts ea

a) $\alpha_1 = \frac{R_d T_1}{p_1} = \frac{(287 \text{ J kg}^{-1} \text{ K}^{-1})(283.15 \text{ K})}{10^5 \text{ Pa}}$

$= \boxed{0.81 \text{ m}^3 \text{ kg}^{-1}}$

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b) $T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{R_d/c_p} = (283.15 \text{ K})\left(\frac{700}{1000}\right)^{.286}$

$= \boxed{255.6 \text{ K}}$

$\alpha_2 = \frac{R_d T_2}{p_2} = \frac{(287)(255.6)}{7 \times 10^4 \text{ Pa}} = \boxed{1.05 \text{ m}^3 \text{ kg}^{-1}}$

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c) $\Delta u = c_v \Delta T$

$= (717 \text{ J kg}^{-1} \text{ K}^{-1})(255.6 - 283.1) \text{ K}$

$= \boxed{-1.97 \times 10^4 \text{ J kg}^{-1}}$

$\Delta h = c_p \Delta T$

$= (1005 \text{ J kg}^{-1} \text{ K}^{-1})(255.6 - 283.1) \text{ K}$

$= \boxed{-2.76 \times 10^4 \text{ J kg}^{-1}}$

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d) $dw = p\,d\alpha$ (adiabatic) $\rightarrow dw = -du$

$\Delta W = -\Delta u = -1.97 \times 10^4 \text{ J kg}^{-1}$ from part (c)

Find mass in 1 km^3 @ $\cancel{700}$ 1000 mb

$m = \frac{V}{\alpha} = \frac{1 \text{ km}^3}{.81 \text{ m}^3 \text{ kg}^{-1}} = \frac{(10^3 \text{ m})^3 / \text{km}^3}{0.81 \text{ m}^3 \text{ kg}^{-1}} = 1.23 \times 10^9$ kg

$\therefore W = m \Delta u = (1.23 \times 10^9 \text{ kg})(-1.97 \times 10^4 \text{ J kg}^{-1}) = \boxed{2.43 \times 10^{13} \text{ J}}$

[3.] (10 pts)

NCB $h = u + p\alpha$

a) Prove dh is exact

Assuming ideal gas, $p\alpha = RT$

$$h = u + RT$$
$$dh = du + RdT$$
$$= c_v dT + RdT$$
$$= (c_v + R)\, dT \quad (= f(T))$$
$$dh = c_p dT$$

Exact, as is $du = c_v dT$.

b) $p_1 = 70$ kPa $\quad p_2 = 100$ kPa

$T_1 = 10°C = 283.15$ K

adiabatic compression $\Rightarrow T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\kappa}$

$$= 283.15 \left(\frac{100}{70}\right)^{\kappa}$$
$$= 313.56 \text{ K}$$

$$\int_1^2 dh = \Delta h = \int_{T_1}^{T_2} c_p\, dT = c_p \Delta T$$
$$= (1005.7 \text{ J kg}^{-1} \text{ K}^{-1})(313.56 - 283.15)\text{K}$$
$$= 3.058 \times 10^4 \text{ J kg}^{-1}$$

4.

Cold case	θ_e simple	251.67
	θ_e Bolton	251.90
Warm case	θ_e simple	362.1
	θ_e Bolton	368.4

$\boxed{5.}$ Specific heat definition

5 pts

$$c = \frac{dq}{dT}$$

For adiabatic, $dq = 0 \Rightarrow c = 0$

For isothermal $dq \to \infty \to$ undefined

$$\lim_{\Delta T \to 0} \frac{dq}{dT} = \infty$$

$\boxed{6.}$ Cruising altitude ~ 10 km (35,000 ft)

a) Assume $T_{sfc} \approx 25°C$

10 pts.

$$\left(\frac{\partial T}{\partial Z}\right) \approx 6.5°C/km$$

$$\Rightarrow T_{10km} = T_{sfc} - \frac{\partial T}{\partial Z} \cdot 10 \text{ km}$$

$$= 25°C - 10 \text{ km} \cdot 6.5 °C/km$$

$$= 25 - 65 = -40°C = 233 K$$

b) Assume $P_{outside} = 25$ kPa

If this air is brought inside, via adiabatic compression, then $T_{in} = T_{out}\left(\frac{P_{in}}{P_{out}}\right)^{\kappa}$

$$= 233 \left(\frac{85}{25}\right)^{.286}$$

$$= 330.6 \text{ K} = 57.6 °C$$

$\therefore$ Air must be cooled.

$\boxed{7.}$ a) Heated air expands

~~8 pts.~~

From the eq. of state, $p\alpha = RT$, we see that, for fixed p, $\alpha \propto T$, so

10 pts

α increases as T increases.

b) When air expands, it cools.

In this case, p decreases in an adiabatic ~~expansion~~ expansion:

$$dq = 0 = c_v dT + p d\alpha$$

$$c_v dT = -p d\alpha = -dw$$

For $d\alpha > 0$ (expanding air), $dT < 0$

Hence the air cools.

Tsonis 4.2: A sample of 100 g of dry air has initial T of 270 K and p = 900 mb. During an isobaric process heat is added and the volume expands by 20% of the initial volume. Estimate (a) final temperature, (b) heat added, and (c) work done against the environment.

4.2 $m = 100\ g$ $\quad p = \text{const}$ $\quad$ dry air

$T = 270\ K$ $\quad V_f = 1.2\ V_i$

$p = 900\ mb$ $\quad \alpha_f = 1.2\ \alpha_i$

10 pts

$$dq = c_v dT + p\,d\alpha$$

$$dQ = mc_v dT + p\,dV = mc_p dT - m\alpha \cancel{dp}^{\,0}$$

$$m(c_p - c_v)\,dT = p\,dV$$

$$mR\,dT = p\,dV$$

$$mR\int_{T_i}^{T_f} dT = \underbrace{p\int_{V_i}^{V_f} dV}_{\text{work}}$$

$$pV = mRT$$

$$p\,dV = mR\,dT$$

$$\Delta T = (T_f - T_i) = \frac{W}{mR}$$

$$= \frac{1550\ J}{(0.1\ kg)(287\ J\ kg^{-1}\ K^{-1})}$$

$$= 54.0\ K$$

(a) $\boxed{T_f = 270 + 54 = 324\ K}$

$$V_i = \frac{mRT_i}{p_i}$$

$$= \frac{(0.1\ kg)(287\ J\ K^{-1} kg^{-1})(270\ K)}{90{,}000\ Pa}$$

$$V_i = 0.0861\ m^3$$

$$V_f = 1.2(0.0861\ m^3) = 0.1033\ m^3$$

(c) $\boxed{\begin{aligned} W &= (90{,}000\ Pa)(.1033 - 0.0861)\,m^3 \\ &= 1550\ J \end{aligned}}$

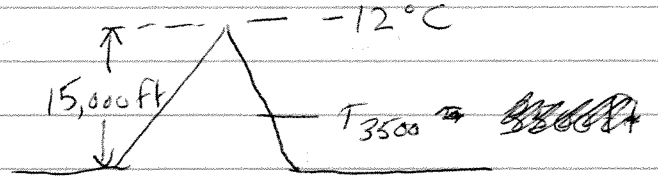
(b) $\Delta Q = mC_v\Delta T + W$

$= (0.1\ kg)(717\ J\ kg^{-1} K^{-1})(54\ K) + 1550\ J$

$= 3872\ J + 1550\ J = \boxed{5422\ J}$

Tsonis 4.5: Assume that you are on the top of a mountain at an altitude of 15,000 ft and there are no clouds above or below. If the T = -12 C, what will the temperture be at 3500 ft altitude?

4.5



Find T_{3500}

Assume dry air, so $\Gamma_d = -9.8\ °C/km$

Difference in altitude $\Delta h = (15{,}000 - 3500)\ ft$

$= 11{,}500\ ft$

$= 11{,}500\ ft * (0.3048\ m/ft)$

$= 3.505\ km$

$T_{3500} = T_{top} + \Gamma_d (\Delta h)$

$= -12\ °C + (-9.8\ °C/km)(-3.505\ km)$

$= -12\ °C + 34.35\ °C$

$= \boxed{22.35\ °C}$

Tsonis 4.7: A sample of dry air has a T = 300 K, a volume = 3 liters, and p = 4 atm. The air undergoes the following changes: (a) warmed at const p to 500 K, (b) cooled at const V to 250 K, (c) cooled at const p to 150 K, and (d) warmed at const V to 300 K. (1) Describe graphically on a (p,V) diagram each of these changes, (2) calculate the total work done.

4.7 dry air

$T = 300$ K

$V = 3$ l

$p = 4$ atm

a) warmed at $p =$ const to 500 K

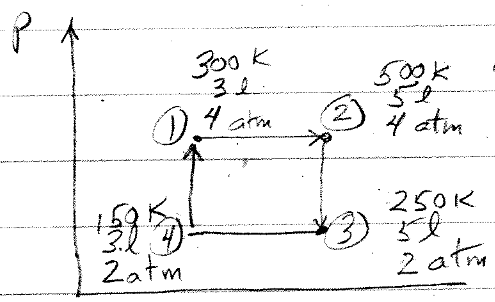
b) cooled @ $V =$ const to 250 K

c) cooled @ $p =$ const to 150 K

d) warmed @ $V =$ const to 300 K

15 pts

$$m = \frac{pV}{RT} = \frac{4(1013.25)(3l)(1m^3/1000l)}{(287\ J\ kg^{-1}K^{-1})(300K)} = 1.41 \times 10^{-4}\ kg = .141\ g$$



initial ①: $p = 4$ atm, $V = 3l$, $T = 300$ K

①→②: $V = \frac{mRT}{p} = \frac{(1.41\times10^{-4}kg)(287)(500)}{4 * 1013.25}$

$= 4.99\ l$

$= 5\ l$

②③ $p \propto T$

$(dQ = C_v dT + p dV \qquad pV = mRT)$

③-④ $V \propto T \qquad V = \left(\frac{150}{250}\right) 5l = 3l$

$$W = p\int_{V_1}^{V_2} dV + p\int_{V_3}^{V_4} dV$$

$= 4\ atm\ (5 - 3\ l) + 2\ atm\ (3 - 5\ l)$

$= +8\ l\cdot atm - 4\ l\cdot atm$

$= 4\ l\cdot atm = 4 * 1013.25 * 100 * 10^{-3}\ m^3\ l^{-1}$

$= \boxed{405.3\ J}$

Petty 5.7 $\theta = 310$ K Find T @ 700, 300, 100 hpa

$$\theta = T\left(\frac{p_0}{p}\right)^{R_d/c_p} \Rightarrow T = \theta\left(\frac{p_0}{p}\right)^{-R_d/c_p}$$

a) $T = 310\left(\frac{1000}{700}\right)^{-0.286} = 279.9$ K

b) $T = 310\left(\frac{1000}{300}\right)^{-0.286} = 219.7$ K

c) $T = 310\left(\frac{1000}{100}\right)^{-0.286} = 160.5$ K

Petty 5.10 $\Delta z = 10$ m $\frac{\Delta T}{\Delta t} = -5 / 10$ hr

$\rho = 1.25$ kg m^{-3}

Assume p = const.

$dq = c_p\, dT - \cancel{\alpha\, dp}^{\,0}$

Find m: mass per unit area

$dQ = m c_p\, dT$

$m = \rho \Delta z$

$= 12.5$ kg m^{-2}

$\dot{Q} = \frac{dQ}{dt} = m c_p \frac{dT}{dt}$

$\dot{Q} = (12.5 \text{ kg m}^{-2})(1005.7 \text{ J kg}^{-1} \text{K}^{-1}) \frac{-5 \text{ K}}{10 \text{ K} \cdot 3600 \text{ s hr}^{-1}}$

$= -1.75 = \text{J m}^{-2} \text{ s}^{-1}$

$= -1.75 \text{ W m}^{-2}$

Petty 5.4

20 pts Compute i) mechanical work and ii) heat added for the following

a) isothermal compression, $T = 15°C$, to $\frac{1}{5}$ orig. volume

i) $W = \int p\,d\alpha \qquad p\alpha = RT \quad \alpha = \frac{RT}{p} \Rightarrow p = \frac{RT}{\alpha}$

$$= RT \int_1^{0.2} d\ln\alpha = RT \ln(0.2)$$
$$= (287)(288)(-1.609)$$
$$\boxed{W = -1.33 \times 10^5 \text{ J kg}^{-1}}$$

ii) $dq = c_v \overset{0\ \text{(isothermal)}}{\cancel{dT}} + p\,d\alpha \Rightarrow dq = dw$

$$\Delta q = w$$
$$= \boxed{-1.33 \times 10^5 \text{ J kg}^{-1}}$$

b) isobaric heating from 0 to 20°C

$dq = c_p\,dT - \cancel{\alpha\,dp}$

ii) $\Delta q = c_p \Delta T = 1005.7(20) = \boxed{2.01 \times 10^4 \text{ J kg}^{-1}}$

i) $dq = c_v\,dT + dw$

$$w = \int dq - \int c_v\,dT = \Delta q - c_v \Delta T$$
$$= 2.01 \times 10^4 - (717)(20)$$
$$= \boxed{5740 \text{ J kg}^{-1}}$$

c) isochoric heating, 0 to 20°C

$$dq = c_v dT + p\cancel{d\alpha}^{\,0}$$

ii) $\Delta q = c_v \Delta T = (717)(20) = 1.43 \times 10^4 \ \text{J kg}^{-1}$

i) $w = \int p\, d\alpha = 0 \quad (d\alpha = 0)$ (isochoric)

d) Adiabatic expansion to 5 time initial volume

$T_i = 20°C$

$$dq = c_v dT + p d\alpha = 0 \quad \text{(adiabatic)}$$

$$c_v dT = -p d\alpha \qquad p\alpha = RT \quad p = \frac{RT}{\alpha}$$

$$c_v dT = -RT \frac{d\alpha}{\alpha}$$

$$\int_{T_i}^{T_f} \frac{dT}{T} = -\frac{R}{c_v} \int_{\alpha_i}^{5\alpha_i} \frac{d\alpha}{\alpha}$$

$$\ln \frac{T_f}{T_i} = -\frac{R}{c_v} \ln 5$$

$$T_f = T_i \exp\left(-\frac{R}{c_v} \ln 5\right)$$

$$= 293 \exp\left(-\frac{287}{717} \cdot 1.609\right) = 153.9 \ \text{K}$$

i) $\int dw = w = -c_v \int_{T_i}^{T_f} dT = -c_v (153.9 - 293.15)$

$= 9.9 \times 10^4 \ \text{J kg}^{-1}$

ii) $\Delta q = 0$ (adiabatic)