Exam 1 (Chaps. 1-6 of the notes)

ATS 541 students: Answer all questions
ATS 441 students: You may delete problem 3 or problem 5

1. [10 pts]

a) Check the following quantities that are conserved when unsaturated air experiences an adiabatic expansion (e.g., by vertical lifting with no mixing)?

_____ dewpoint temperature (T_d)
_____ enthalpy (h)
_____ entropy (s)
_____ equivalent potential temperature (θ_e)
_____ mixing ratio (r_v)
_____ potential temperature (θ)
_____ relative humidity (f)
_____ water vapor pressure (e)
_____ wet-bulb potential temperature (θ_w)
_____ wet-bulb temperature (T_w)

a) Check the following quantities that are conserved when saturated air is lifted adiabatically (no mixing, and all condensed water stays with the parcel)?

_____ dewpoint temperature (T_d)
_____ enthalpy (h)
_____ entropy (s)
_____ equivalent potential temperature (θ_e)
_____ mixing ratio (r_v)
_____ potential temperature (θ)
_____ relative humidity (f)
_____ water vapor pressure (e)
_____ wet-bulb potential temperature (adiabatic, θ_w)
_____ wet-bulb temperature (T_w)

2. [30 pts]
Consider a parcel with the following thermodynamic values:

$p = 900$ hPa

$T = 25$ °C

$r_v = 10$ g kg^{-1}

Using the skew-T diagram on the following pages, find the following parameters graphically. Also write the equation that would be used to determine the *most precise* values.

a) saturation point T and p values (T_{sp} and p_{sp}; give equation for T_{sp} only)

b) saturation mixing ratio

c) potential temperature

d) equivalent potential temperature

f) wet-bulb potential temperature

g) dew-point temperature

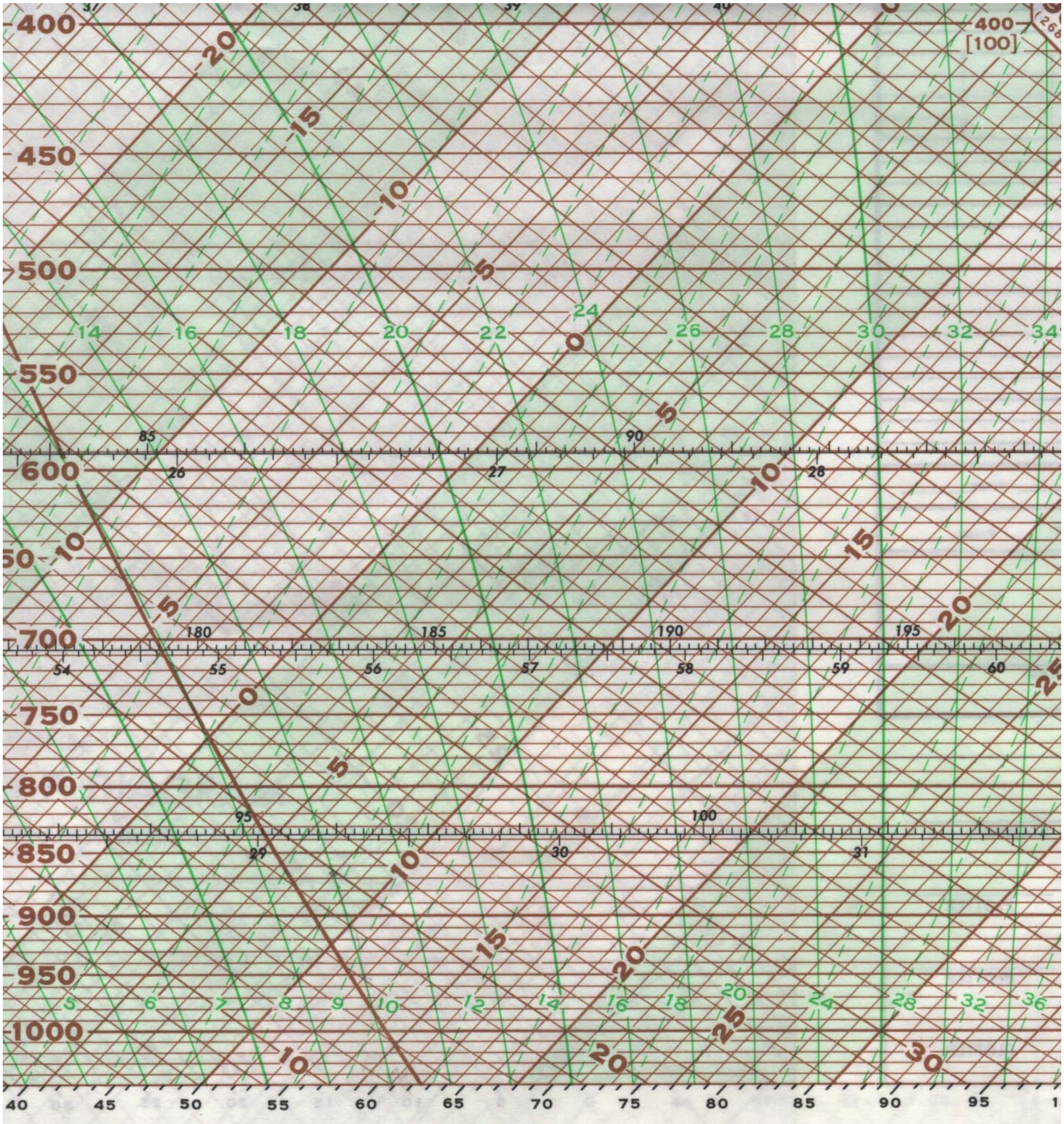
h) relative humidity

i) vapor pressure

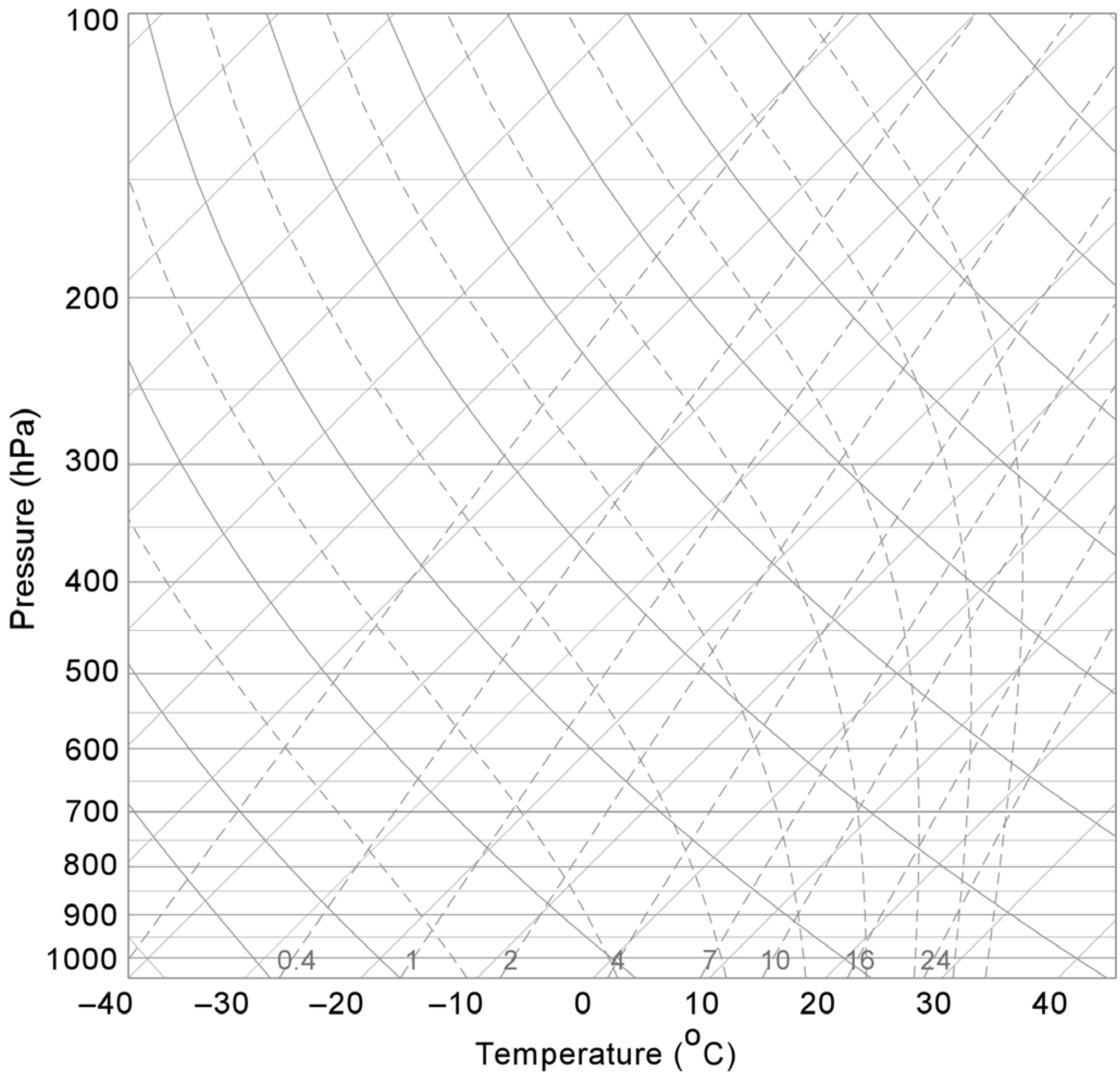
j) the temperature of a parcel that ascends adiabatically to 600 mb

k) the adiabatic mixing ratio of the parcel at 600 mb

Show all work on the skew-T diagrams on the next two pages.



400
450
500
550
600
700
750
800
850
900
950
1000
400
[100]
14
16
18
20
22
24
26
28
30
32
34
5
6
7
8
9
10
12
14
16
18
20
24
28
32
36
85
90
26
27
28
180
185
190
195
54
55
56
57
58
59
60
95
100
29
30
31
40
45
50
55
60
65
70
75
80
85
90
95


Pressure (hPa)
100
200
300
400
500
600
700
800
900
1000
0.4
1
2
4
7
10
16
24
–40
–30
–20
–10
0
10
20
30
40
Temperature (°C)

3. [15 pts]
What is the volume, in m^3, of 5 kg of dry air at 300 mb and -40 °C?

4. [10 pts]
A parcel of moist air has a total pressure of 975 hPa and a temperature of 15 °C. If the mixing ratio is 1.8 g kg^{-1}, what are the water vapor pressure and the virtual temperature?

5. [15 pts]
An isolated rain drop that is evaporating into air with a temperature of 18 °C has a water surface temperature of 12 °C.
a) Find (using a skew-T) the mixing ratio of the air.
b) Indicate what equation you would use to solve this problem precisely.

6. [20 pts]

a) Convert the First Law of Thermodynamics from the T, α form ($dq = c_v dT + p d\alpha$) to the T, p form. Show all steps.

b) Derive potential temperature (θ) from this form ($dq = c_p dT - \alpha dp$) of the First Law

c) Explain why a correction factor in the exponent is used when water vapor is present.

Equations and constants

$p\alpha = (R^*/M)T = RT$ $p = \rho_m R_d T_v$. $e = \rho_v R_v T$

$dq = c_v dT + p d\alpha$. $dq = c_p dT - \alpha dp$. $dq = T ds = du + p d\alpha$.

$T\alpha^{\eta-1} = \text{const}$ $(T^{c_v}\alpha^{R_d} = \text{const})$
$Tp^{-\kappa} = \text{const}$ $(T^{c_p}p^{-R_d} = \text{const}$ $\kappa = R_d/c_p$ and $\eta = c_p/c_v$.
$p\alpha^{\eta} = \text{const}$ $(p^{c_v}\alpha^{c_p} = \text{const})$

$c_{pm} = c_{pd}(1+0.887r_v)$ $R_m = R_d(1+0.608r_v)$.

$$\theta_e = \theta \exp\left(\frac{L_{vl} r_{vs}}{c_p T_{sp}}\right) \qquad \theta_e = \theta \exp\left(\frac{2675 r_{vs}}{T_{sp}}\right)$$

$$ds \equiv \frac{dq_{rev}}{T} \qquad \theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}(1-0.28 r_v)}$$

$f \equiv u - Ts$ $g = u - Ts + p\alpha$

$dp/dT = \Delta s/\Delta\alpha = -\Delta H_{fusion}/(T\Delta\alpha)$ $d\ln e_s/dT = L_{vl}/(R_v T^2)$

$e_s(T) = Ae^{-B/T}$ $A = 2.53\times10^8$ kPa, $B = 5.42\times10^3$ K

$r_v = m_v/m_d = \varepsilon e / [p-(1-e)e] \cong \varepsilon e/p$ $q_v = \varepsilon e / [p-(1-e)e]$ $\varepsilon = 0.622$

$f = r_v / r_{vs}(T,p) = r_{vs}(T_d,p)/r_{vs}(T,p) \approx e/e_s(T)$

$T_v \cong T(1+0.61r_v)$ $$T_d = T_d(r_{vs},p) = \frac{B}{\ln\left(\frac{A\varepsilon}{r_v p}\right)} = f(r_v,p)$$

$T_{iw} = T - (L_{lv}/c_p)[(\varepsilon/p)Ae^{-B/T_w} - r_v]$, $T_{ie} = T + L_{lv}r_v/c_p$.

$$T_{sp} = \frac{2840}{3.5\ln T - \ln e - 4.805} + 55.$$

$\chi = \rho_m [r_{vs}(T_{sp},p_{sp}) - r_{vs}(T_{sat},p)_{\theta_{e=const}}]$

$$\theta_e = T_K \left(\frac{1000}{p}\right)^{0.2854(1-0.28r_v)} \exp\left[\left(\frac{3.376}{T_{sp}} - 0.00254\right) r_v (1 + 0.81 r_v)\right]$$

c_v = 717 J K^{-1} kg^{-1}
c_p = 1005.7 J K^{-1} kg^{-1}
c_{wv} = 1463 J K^{-1} kg^{-1}
c_{wp} = 1952 J K^{-1} kg^{-1}
R_d = 287 J kg^{-1} K^{-1} R_v = 461.5 J kg^{-1} K^{-1}
L_{vl} = 2.50 x 10^6 J kg^{-1} (0 ^{o}C) latent heat of condensation (function of T)
L_{vl} = 2.25 x 10^6 J kg^{-1} (100 ^{o}C)
L_{il} = 3.34 x 10^5 J kg^{-1} latent heat of melting
L_{vi} = 2.83 x 10^6 J kg^{-1} (0 ^{o}C) latent heat of deposition
L_{vl} = (2.501 - aT_c) x 10^6 J kg^{-1}, where a = 0.00237 °C^{-1} and T_c is the dry bulb temperature in °C.

Table 5.1. Saturation vapor pressures over water and ice, and latent heats of condensation and deposition.

T (°C)	e_s (Pa)	e_i (Pa)	L_{vl} (J kg^{-1}) x 10^6	L_{vi} (J kg^{-1}) x 10^6
-40	19.05	12.85	2.603	2.839
-35	31.54	22.36		
-30	51.06	38.02	2.575	2.839
-25	80.90	63.30		
-20	125.63	103.28	2.549	2.838
-15	191.44	165.32		
-10	286.57	259.92	2.525	2.837
-5	421.84	401.78		
0	611.21	611.15	2.501	2.834
5	872.47		2.489	
10	1227.94		2.477	
15	1705.32		2.466	
20	2338.54		2.453	
25	3168.74		2.442	
30	4245.20		2.430	
35	5626.45		2.418	
40	7381.27		2.406	