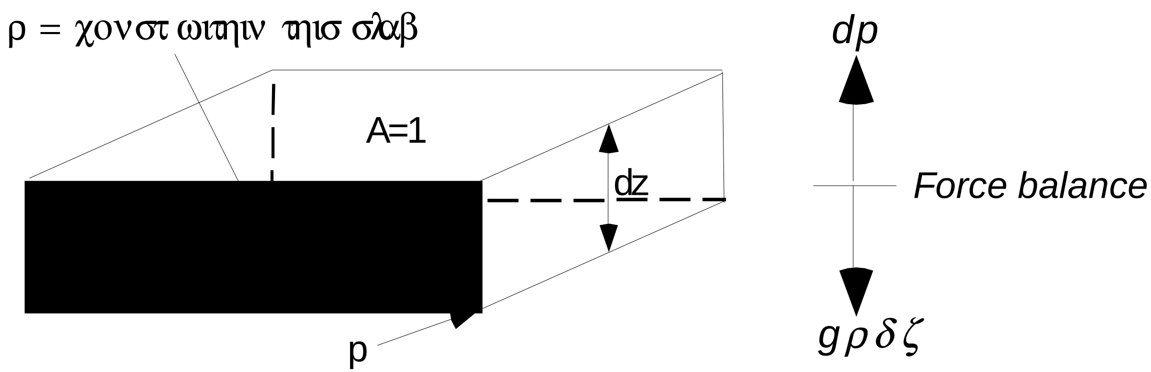
Chapter 8

Atmospheric Statics and Stability

1. The Hydrostatic Equation

- HydroSTATIC – dw/dt = 0!
- Represents the balance between the upward directed pressure gradient force and downward directed gravity.



υπωαρδ πρεσσυρε γραδιεντ φορχε = δοωνωαρδ φορχε βψ γραϖιτψ

- p=F/A. A=1 m^2, so upward force on bottom of slab is p, downward force on top is p-dp, so net upward force is dp.
- Weight due to gravity is F=mg=ρgdz
- Force balance: **dp/dz = -ρg**

2. Geopotential

- Like potential energy. It is the work done on a parcel of air (per unit mass, to raise that parcel from the ground to a height z.
- $d\phi \equiv g\,dz$, so

$$\phi(z) = \int_0^z g\,dz$$

- Geopotential height – used as vertical coordinate often in synoptic meteorology.
- $Z \equiv \phi(z)/g_o$ (where g_o is 9.81 m/s^2).
- Note: Since gravity decreases with height (only slightly in troposphere), geopotential height Z will be a little less than actual height z.

3. The Hypsometric Equation and Thickness

- Combining the equation for geopotential height with the hydrostatic equation and the equation of state $p = \rho R_d T_v$,

$$Z_2 - Z_1 = -\frac{R_d}{g_0} \int_{p_1}^{p_2} T_v \, d \ln p$$

- Integrating and assuming a mean virtual temp (so it can be a constant and pulled outside the integral), we get the hypsometric equation:

$$Z_2 - Z_1 = \frac{R_d \overline{T}_v}{g_0} \ln\left(\frac{p_1}{p_2}\right)$$

- For a given mean virtual temperature, this equation allows for calculation of the thickness of the layer between 2 given pressure levels.

- For two given pressure levels, the thickness is lower when the virtual temperature is lower, (ie., denser air).

- Since thickness is readily calculated from radiosonde measurements, it provides an excellent forecasting tool.

- For example, the thickness is one of the most commonly cited rain/snow forecast tools.

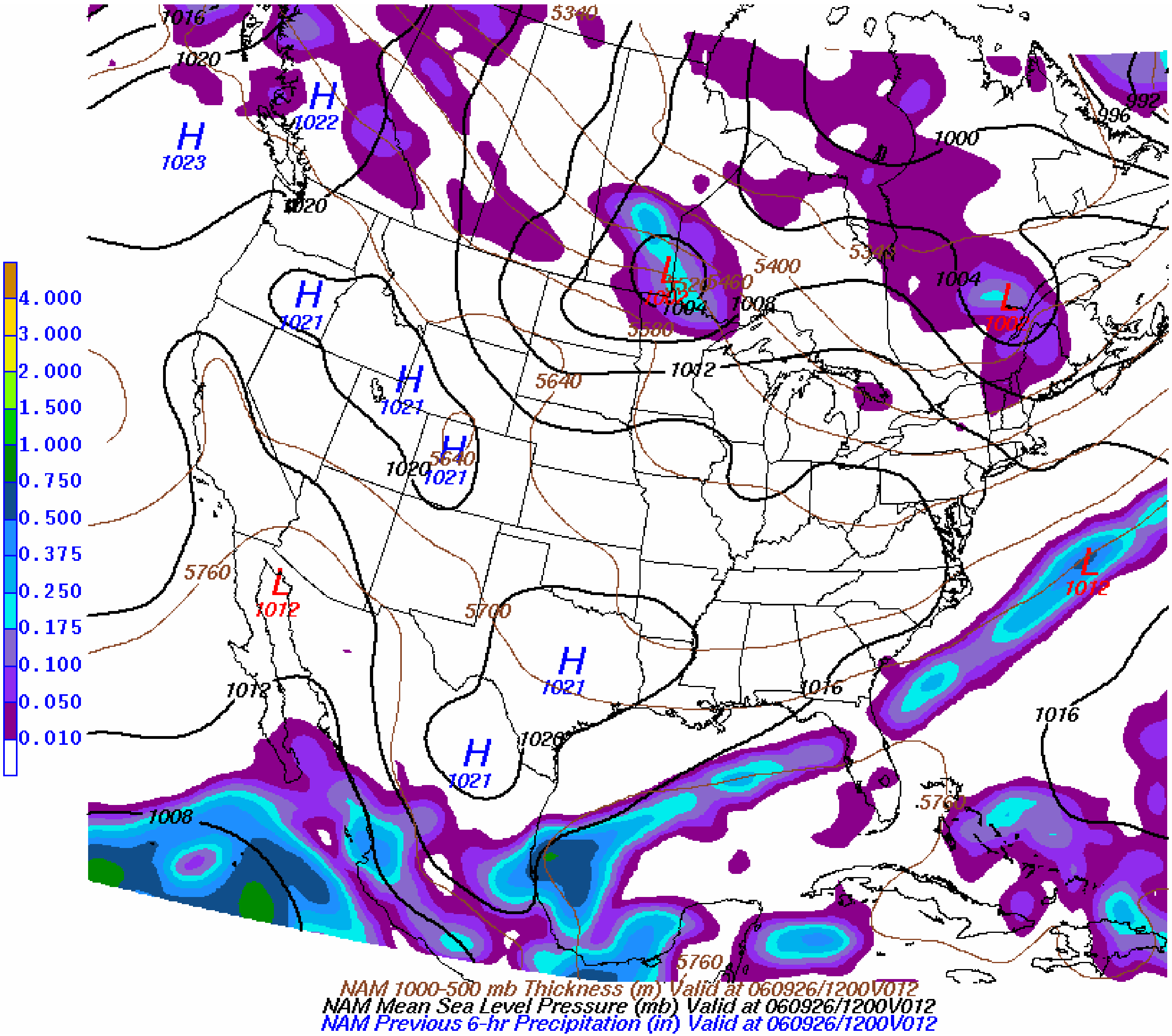
- In Alabama, critical thickness for snow: 1000-850 mb: 1300 m

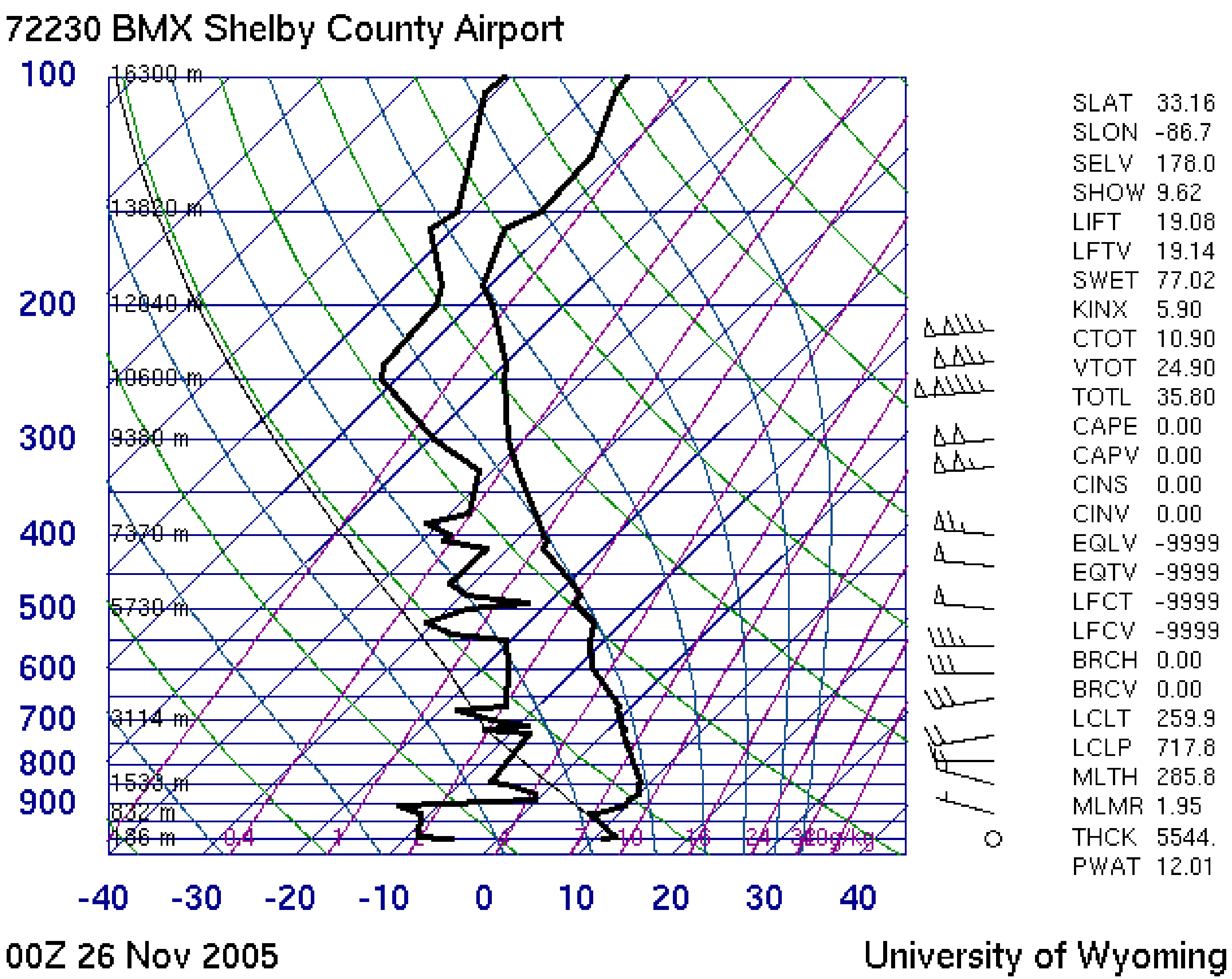
  1000-500 mb: 5440 m

- Thickness changes (without significant air mass change) are useful in forecasting high temps.

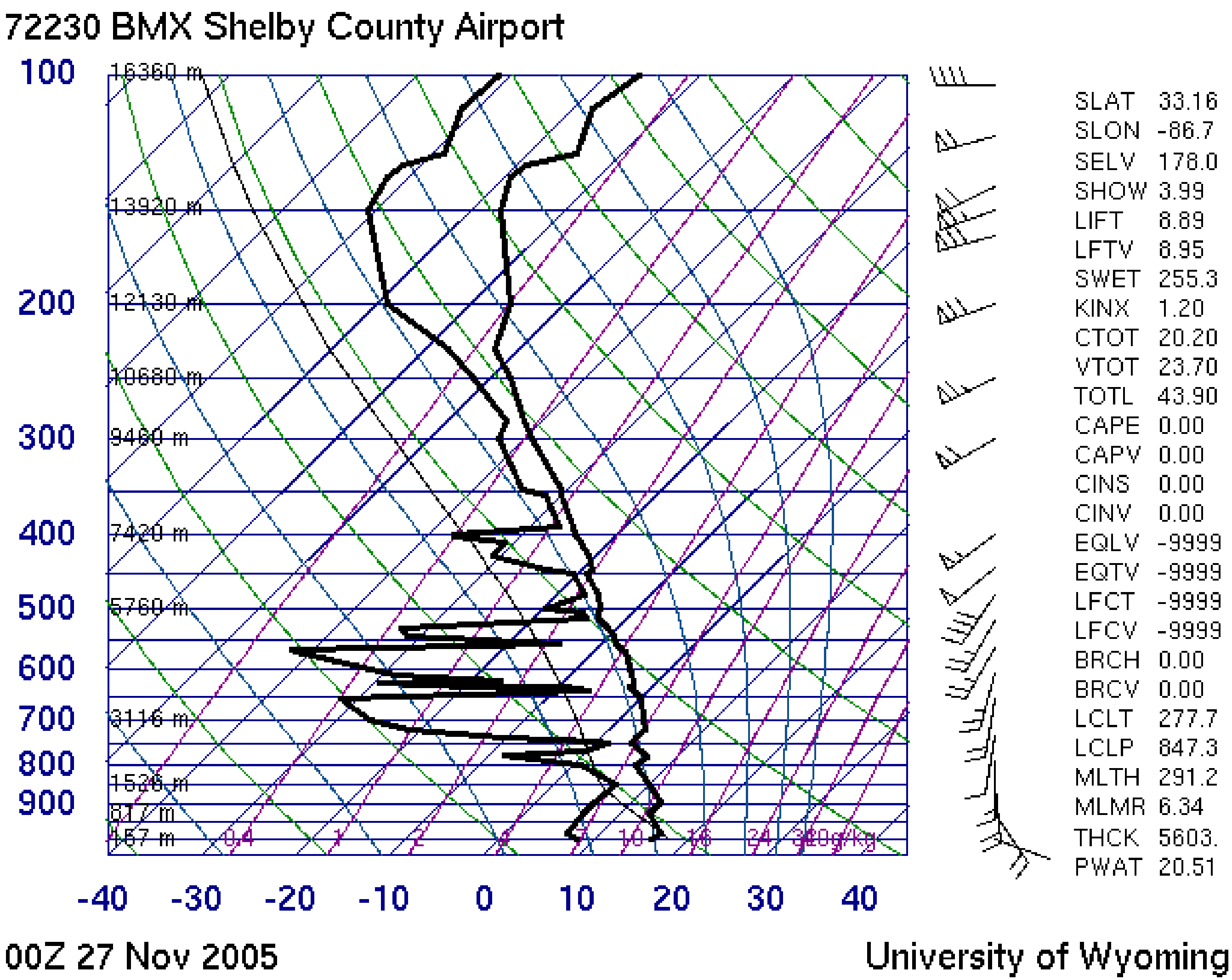
- For example, for the 1000-500 mb layer, at a mean virtual temp of 273 K, each 1 K temp increase represents a thickness increase of about 20 m (or 2 dekameters).

- Now, since thickness is usually represented in dekameters, and 1 K is about 2 degrees Farenheit, a technique sometimes used in the NWS (eg., Tobias 1991) states that a 1 dekameter change in 1000-500 mb thickness will represent a 1 degree change in the high temperature.


4.000
3.000
2.000
1.500
1.000
0.750
0.500
0.375
0.250
0.175
0.100
0.050
0.010
NAM 1000-500 mb Thickness (m) Valid at 060926/1200V012
NAM Mean Sea Level Pressure (mb) Valid at 060926/1200V012
NAM Previous 6-hr Precipitation (in) Valid at 060926/1200V012



1000-500 mb thickness: 5544 m. High temp: 56



1000-500 mb thickness: 5603 m High temp: 67

4. Scale height and atmospheric pressure profile

- Now, the term in the hypsometric equation $R_d T_v/g$ is defined as the atmospheric scale height, and is equal to 29.3 T_v.

- Again combining the hydrostatic equation with the equation of state,

$$\frac{dp}{p} = -\frac{g}{R_d T_v} dz$$

- Integrating from p_o,z_o to p_1,z_1

$$p = p_o \exp\left(-\frac{g}{R_d \overline{T_v}}(z - z_o)\right),$$

- Or, in terms of scale height, $p = p_o \exp(-z/H)$

- So, pressure decreases exponentially with height in the atmosphere, and this decrease occurs more quickly in colder air.

- Note that cold air is typically marked by upper-level troughs.

5. Idealized atmospheres

- *Homogeneous atmosphere* (constant density)

  Can only occur very close to the surface (lowest 1 m), usually over asphalt, etc. Causes mirages. (Ie., car hood)

$$-p_o = \int_{p_o}^{0} dp = -\rho g \int_{0}^{H} dz = -\rho g H.$$

Then $$H = \frac{p_o}{\rho g} = \frac{R_d T_v}{g},$$ $(p = \rho R_d T_v)$.

For T_v = 273 K, H = (287)(273)/(9.81) = 7.99 km.

The lapse rate corresponding to the homogeneous atmosphere is obtained as follows. If we differentiate the equation of state [$p=\rho R_d T_v$] we obtain

$$\frac{\partial p}{\partial z} = \rho R_d \frac{\partial T_v}{\partial z} = -\rho g \qquad [\rho = \text{const}] \qquad \frac{\partial T_v}{\partial z} = \frac{-g}{R_d}$$

Lapse rate in this atmosphere is dT/dz = -34.1 K/km!

- *Isothermal atmosphere* (constant temp)

Occurs over limited regions of the troposphere

$p = p_o \exp(-z/H)$ works here, where $H = R_d T_v / g$, but here, instead of using the mean virtual temperature, it is constant.

$$\frac{dp}{dz} = \frac{-pg}{R_d T_v} \qquad \int_{p_o}^{p} \frac{dp}{p} = \int_{0}^{z} \frac{-g}{R_d T_v} dz$$

Thus, $\ln(p/p_o) = -gz/(R_d T_v) = -z/H$. Taking the exponential of each side gives

$p = p_o \exp(-z/H)$,

Constant lapse rate atmosphere

The virtual temperature profile in this case is assumed to be

$$T_v(z) = T_{vo} - \gamma z,$$

where γ –™T$_\varpi$/™ζ Αγαι , ωε υσε τηε ηψδροστατιχ εθυατιον, ρεαρρανγε ανδ ιντεγρατε το γετ

or ,

φ .

8.3.4 *Dry adiabatic atmosphere*

This is a special case of the constant lapse rate atmosphere considered above. Poisson's equation applies:

$$\Gamma_d = -g/c_p = -9.76 \text{ K km}^{-1}.$$

- *U. S. Standard Atmosphere*

Based on an average atmosphere temperature profile at 40 °N

T_o = 288 K, p_o = 1013.25 hPa

Dry, hydrostatic

dT/dz = -6.5 K/km from sea level to 10.77 km above MSL

T = 216.5 K (constant) above 10.77 km

Exam 1

- ATS 441

High - 93

Average - 82

- ATS 541

High - 93

Average - 79.5

6. Static Stability

- Defines the resistance of air parcels to vertical displacements due to buoyancy.

- In a stable atmosphere, a parcel displaced will experience an acceleration back towards its equilibrium position.

- In an unstable atmosphere, a parcel displaced up/down will accelerate in the direction of the displacement.

- Buoyancy is evaluated by Archimedes' Principle, and

$$\frac{dw}{dt} = g\frac{(\rho_o - \rho')}{\rho_o} = gB$$

and through the equation of state, (assuming p=p')

$$\frac{dw}{dt} \cong g\frac{T_v' - T_{vo}}{T_{vo}} = gB$$

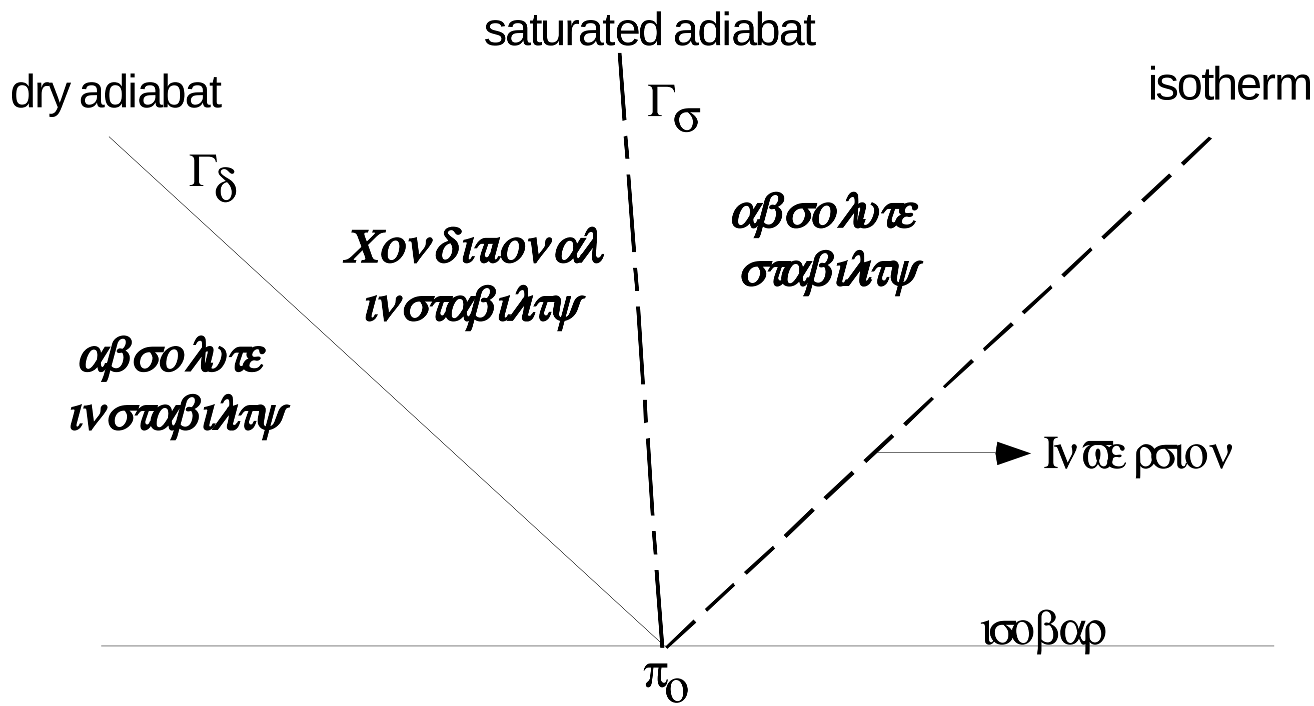
- So, if an air parcel is warmer than its environment, it will accelerate upward, and if it is colder than its environment, it will accelerate downward.

- Now, if a dry air parcel is displaced upward, it cools at the dry adiabatic lapse rate, $dT/dz = -9.8$ K/km.

- If a saturated air parcel is displaced upward, it cools (more slowly, due to the offsetting effect of latent heat) at the saturated adiabatic lapse rate.

- In either case, if a rising air parcel cools more quickly than its environment, it will immediately become cooler than the environment and accelerate back down. This would be stable air.

- If a rising air parcel cools slower than the environment, it will become warmer than the environment and accelerate upward. This is unstable air.



- absolute stability: $\gamma < \Gamma_s$
- absolute instability: $\gamma > \Gamma_d$
- conditional instability: $\Gamma_s < \gamma < \Gamma_d$
- neutral (subsaturated): $\gamma = \Gamma_d$
- neutral (saturated): $\gamma = \Gamma_s$

Special cases

7. Stability and potential temperature

<u>*Relation between lapse rate of T and θ:*</u>

Poisson's Eq. is

[illegible]

Application of the log differential to Poisson's Eq. gives

[illegible].

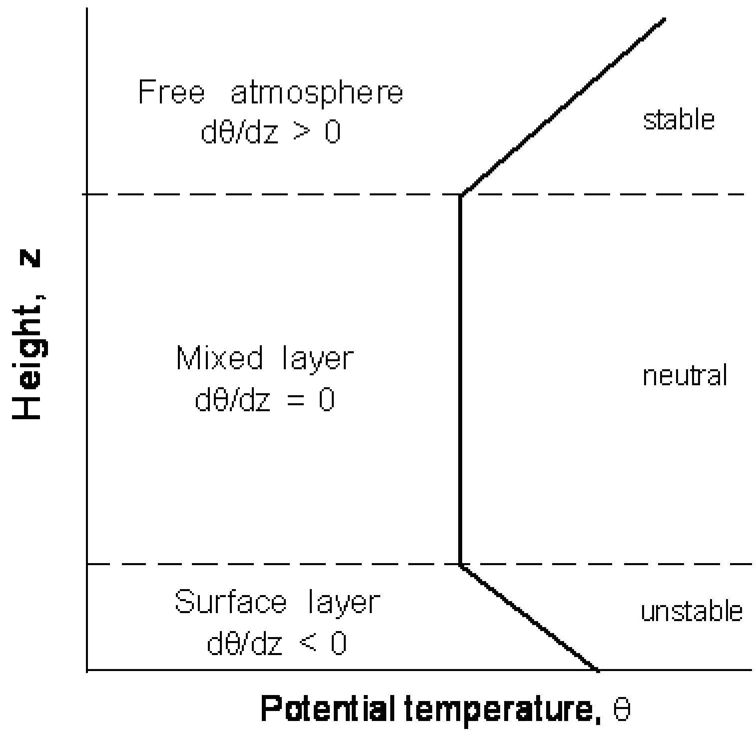
We now take the partial derivative w.r.t. height to get

[illegible]

Substitution of the equation of state and hydrostatic eq. in the last term gives

[illegible],

- When analyzing stability (neglecting moisture), one can look at the vertical gradient in potential temperature $d\theta/dz$.
- When $d\theta/dz > 0$, the air is stable.
- When $d\theta/dz < 0$, the air is unstable
- When $d\theta/dz = 0$, the air is neutral



1. Brunt-Vaisala frequency and gravity waves

• When air is stable (ie., it accelerates back towards its equilbrium position upon vertical displacement), an oscillation, similar to simple harmonic motion, is set up.

• The more stable the air, the higher the frequency will be.

• The frequency is known as the Brunt-Vaisala frequency, N.

• It is this type of oscillation which sets up gravity waves in the atmosphere.

Acceleration: $$\frac{dw}{dt} = \frac{d^2z}{dt^2} = -\left(\frac{g^2}{T_v}\right)(\gamma - \Gamma)z$$

Harmonic oscillator $$\frac{d^2z}{dt^2} + kz = 0 \qquad k = \frac{g^2}{T_v}(\gamma - \Gamma) = \omega^2 = \left(\frac{2\pi}{\tau}\right)$$

Solution: Z(t) = Asin(ωt); period of motion: $$\tau = \frac{2\pi}{g}\left(\frac{T_v}{(\gamma - \Gamma)}\right)^{\frac{1}{2}}$$

Example: Assume the following atmospheric quantities: T = 0 □Χ(δρψ ατμοσπηερε) γ = -3 K km^{-1}. Since the atmosphere is dry, $\Gamma_i = \Gamma_d$ = -9.8 K km^{-1}. Find τ, and assuming that the amplitude of the wave is 500 m, find the maximum vertical velocity within the wave.

$$\tau = 2\pi[T_v/g(\Gamma_v-\gamma)]^{1/2} = 2\pi[273/9.8(-3+9.8)x10^{-3}]^{1/2}$$
$$= 402 \text{ s.}$$

The maximum velocity is found by differentiating $z(t) = A\cdot\sin(\omega t)$

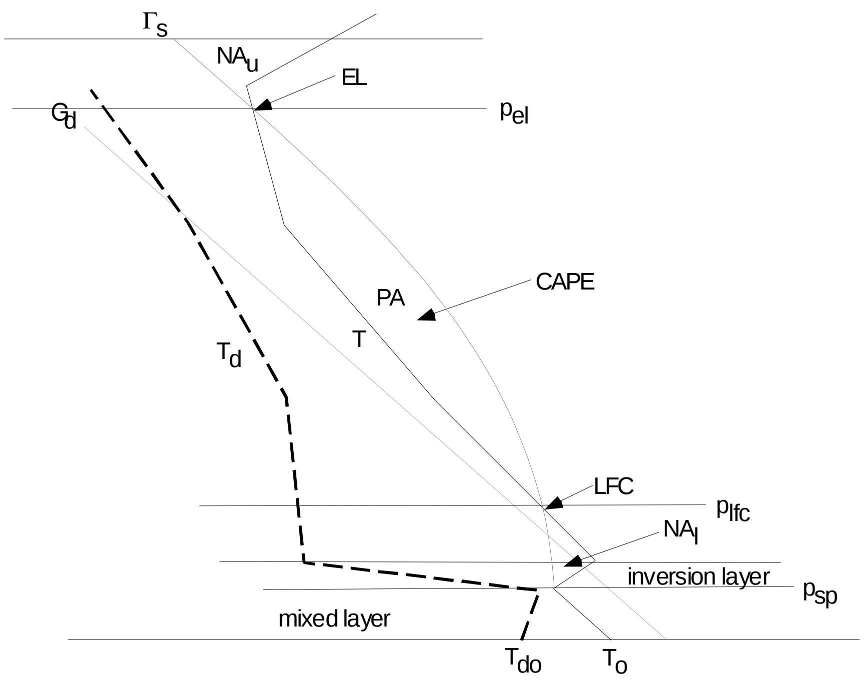
$$w_{max} = dz/dt = A\omega = A(2\pi/\tau) = 500(2\pi/402) = 7.8 \text{ m s}^{-1}!$$

1. The parcel method

• If an unsaturated parcel of air is forced upward from low-levels, it will initially cool at the dry adiabatic lapse rate.

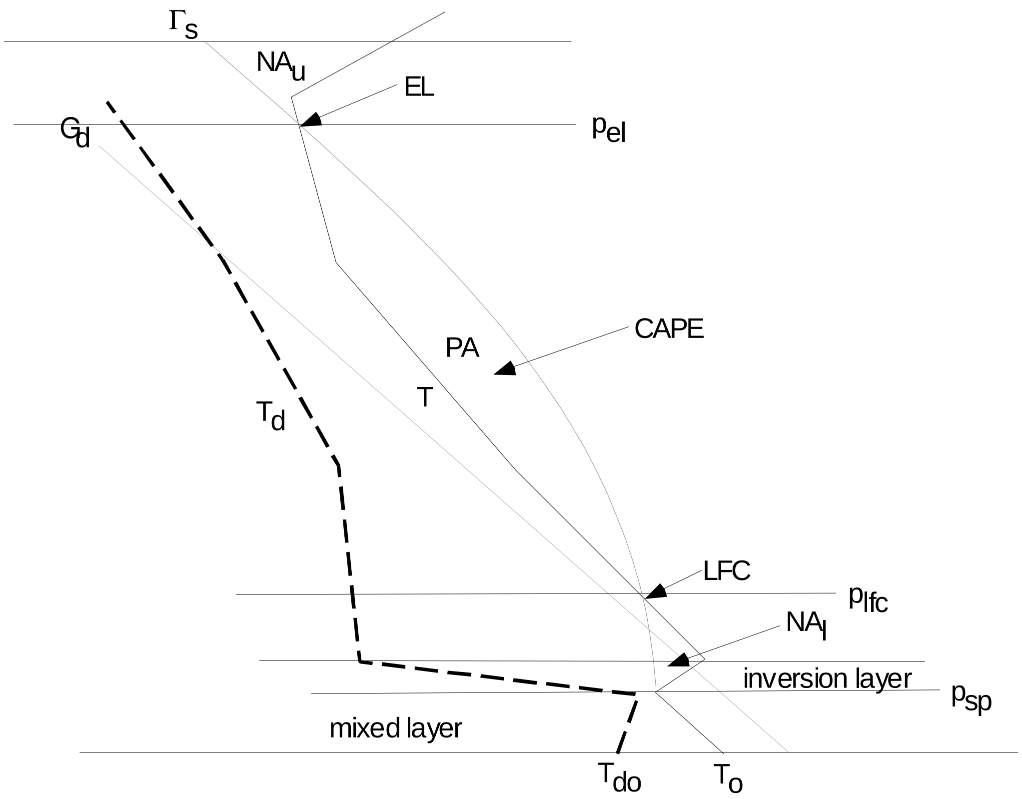
• If it reaches its lifted condensation level (LCL), the air cools at the slower moist adiabatic lapse rate.

• Once the air becomes warmer than its environment, it requires no additional lift (convergence, etc.), and will begin to accelerate upward due to buoyancy. This occurs at the level of free convection (LFC).



• Eventually, often near the tropopause, the air parcel will become colder than its environment again, halting its upward acceleration. This point is known as the equilibrium level (EL).

• The area on a skew T-ln p sounding between the temperature profile of a rising parcel and that of the environment is known as positive area (PA) or negative area (NA), depending on whether the parcel is warmer (PA) or colder (NA) than the environment.



10. CAPE, CIN, and LI

- The NA at the bottom of the sounding indicates just how much forcing is required to reach the LFC. So, it is known as the Convective Inhibition, or CIN. If the CIN is too high, storms can not develop.

- The PA on the sounding is known as the Convective Available Potential Energy, or CAPE. It may be given by

$$CAPE = g\int_{z_{LFC}}^{z_{EL}} \frac{\theta' - \theta_o}{\theta_o} dz \qquad\qquad CAPE = R_d \int_{p_{LFC}}^{p_{EL}} (T_v' - T_{vo})\, d\ln p$$

- CAPE is in units of m^2/s^2, or in J/kg. CAPE varies widely, but on a typical thunderstorm day is 1000-4000 J/kg.

- Since CAPE is related to potential energy per unit mass, one can use the CAPE to determine the maximum possible updraft speed. By equating $KE=1/2\ w^2$ to CAPE, the maximum updraft is $w_{max} = (2*CAPE)^{1/2}$.

- Another measure of stability is the lifted index (LI). It is simply the difference in the temperature of the parcel and the environment at 500 mb. Negative values are unstable, and typical storm values are -3 to -5.

Other forms of CAPE

Modified Convective Available Potential Energy that includes the water vapor and condensed water virtual effects

$$MCAPE = g \int_{z_{LCL}}^{z_{EL}} \left(\frac{\theta_v' - \theta_{vo}}{\theta_{vo}} - r_{vsa} \right) dz$$

Mixed layer CAPE (MLCAPE)

Most Unstable CAPE (MUCAPE)

CAPE be used used to estimate the maximum updrafts in cumulus clouds:

$w_{max} = (2 \bullet CAPE)^{1/2}$

(discuss how this is derived and at what level this would occur)

Lifted Index:

$$LI = T_o(500\text{ mb}) - T_{ad}(500\text{ mb}) \qquad \text{(lifted index)} \qquad [°C]$$

Total totals index:

$$TT = T(850\text{ mb}) + T_d(850\text{ mb}) - 2 \cdot T(500\text{ mb}) \qquad [°C]$$

K index:

$$K = T(850\text{ mb}) - T(500\text{ mb}) + T_d(850\text{ mb}) - (T-T_d)(700\text{ mb}) \qquad [°C]$$

Showalter index:

$$S = T_o(500\text{ mb}) - T_{ad850}(500\text{ mb}) \qquad [°C]$$

where T_{ad850} is the temperature along a psuedo adiabat that passes through the T_{sp} of a parcel that begins ascent with the environmental values of T and T_d at 850 mb.

Question: What are the advantages and disadvantages of using an integrated measure of static stability such as CAPE over the layer estimators such as LI, TT, and K?

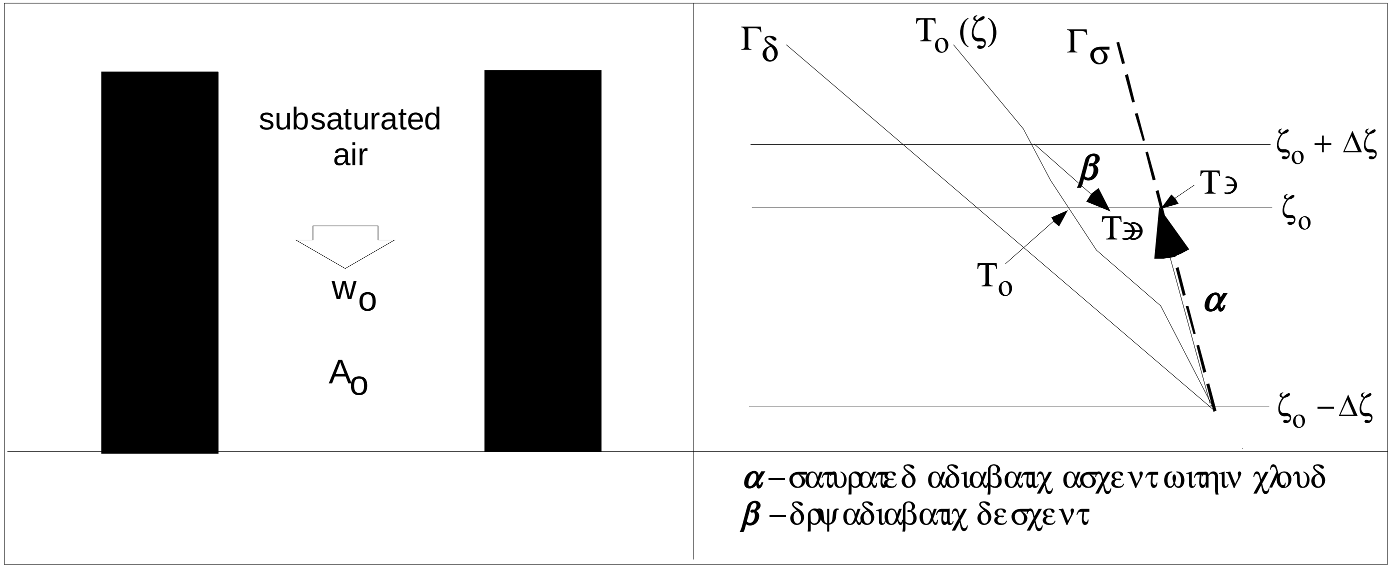
Let’s take a look:

Soundings:
http://www.rap.ucar.edu/weather/upper/

Sounding analysis:
http://www.spc.noaa.gov/exper/

University of Wyoming:
http://weather.uwyo.edu/upperair/sounding.html

8.3 The layer method for analyzing vertical stability



Conservation of mass dictates $\rho A_o w_o = \rho A' w'$,
where A_o, A_c are the areas of the environment and cloud parcel, respectively, and w_e, w_o the vertical motion in each domain.

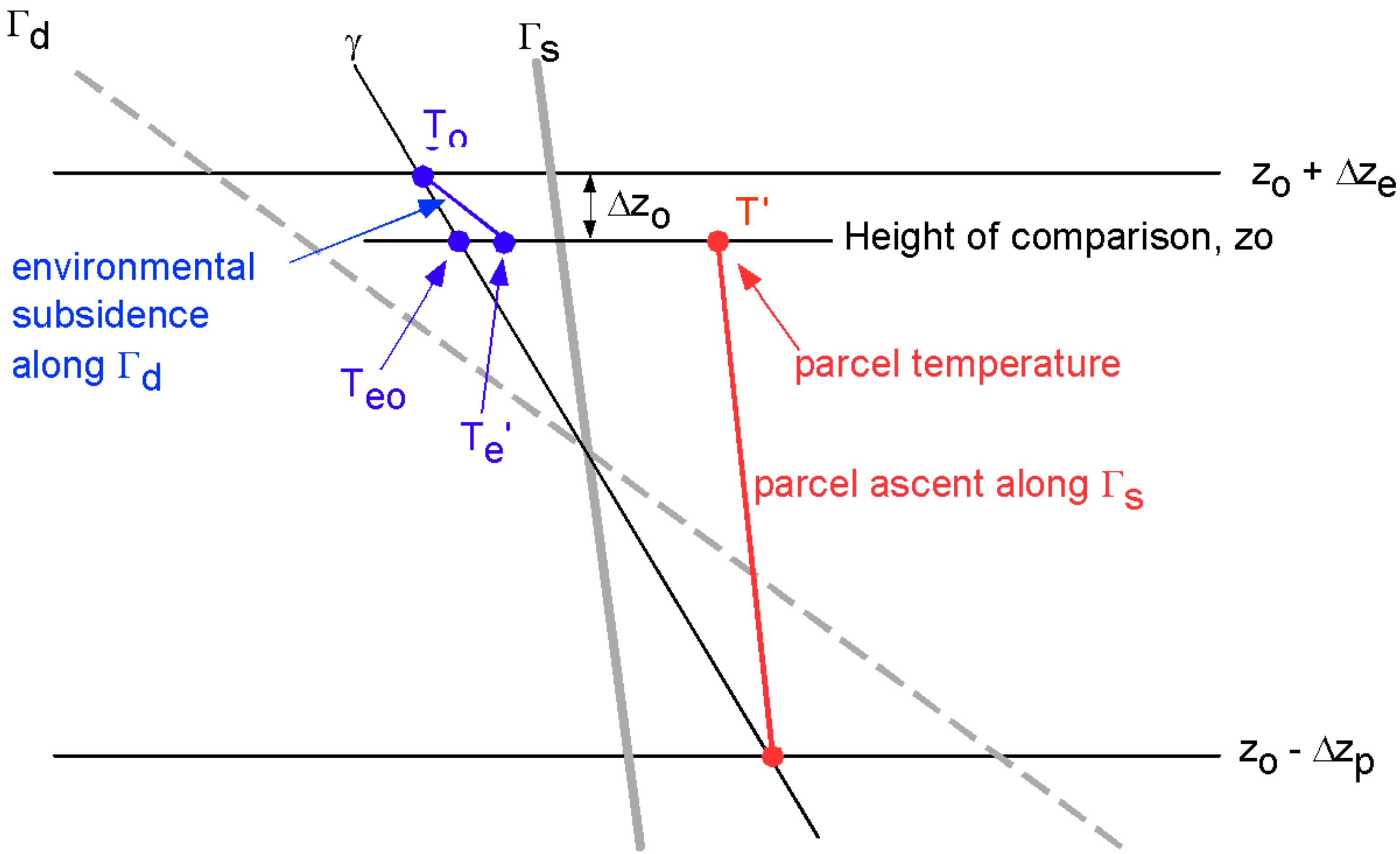
The above can be rewritten as (since w = dz/dt)

$$A_o/A' = w'/w_o = \Delta z'/\Delta z_o \text{ (after a finite time } \Delta t)$$

$$T' = T_{eo} + \gamma\Delta z_p - \Gamma_s\Delta z_p = T_{eo} + (\gamma-\Gamma_s)\Delta z_p$$

$$T_e' = T_{eo} - \gamma\Delta z_e + \Gamma_d\Delta z_e = T_{eo} + (\Gamma_d-\gamma)\Delta z_e$$

$$\Delta T = T_p - T_e = (\gamma-\Gamma_s)\Delta z_p - (\Gamma_d-\gamma)\Delta z_e = [(\gamma-\Gamma_s) - A'/A_e(\Gamma_d-\gamma)]\Delta z_p$$



The new stability criteria for the slice method are:

$$(\gamma - \Gamma_s) > A'/A_e(\Gamma_d - \gamma) \qquad \text{unstable}$$

$$(\gamma - \Gamma_s) = A'/A_e(\Gamma_d - \gamma) \qquad \text{neutral}$$

$$(\gamma - \Gamma_s) < A'/A_e(\Gamma_d - \gamma) \qquad \text{stable}$$

Convective instability

Synoptic or mesoscale vertical motion can modify the static stability (refer to the layer method)

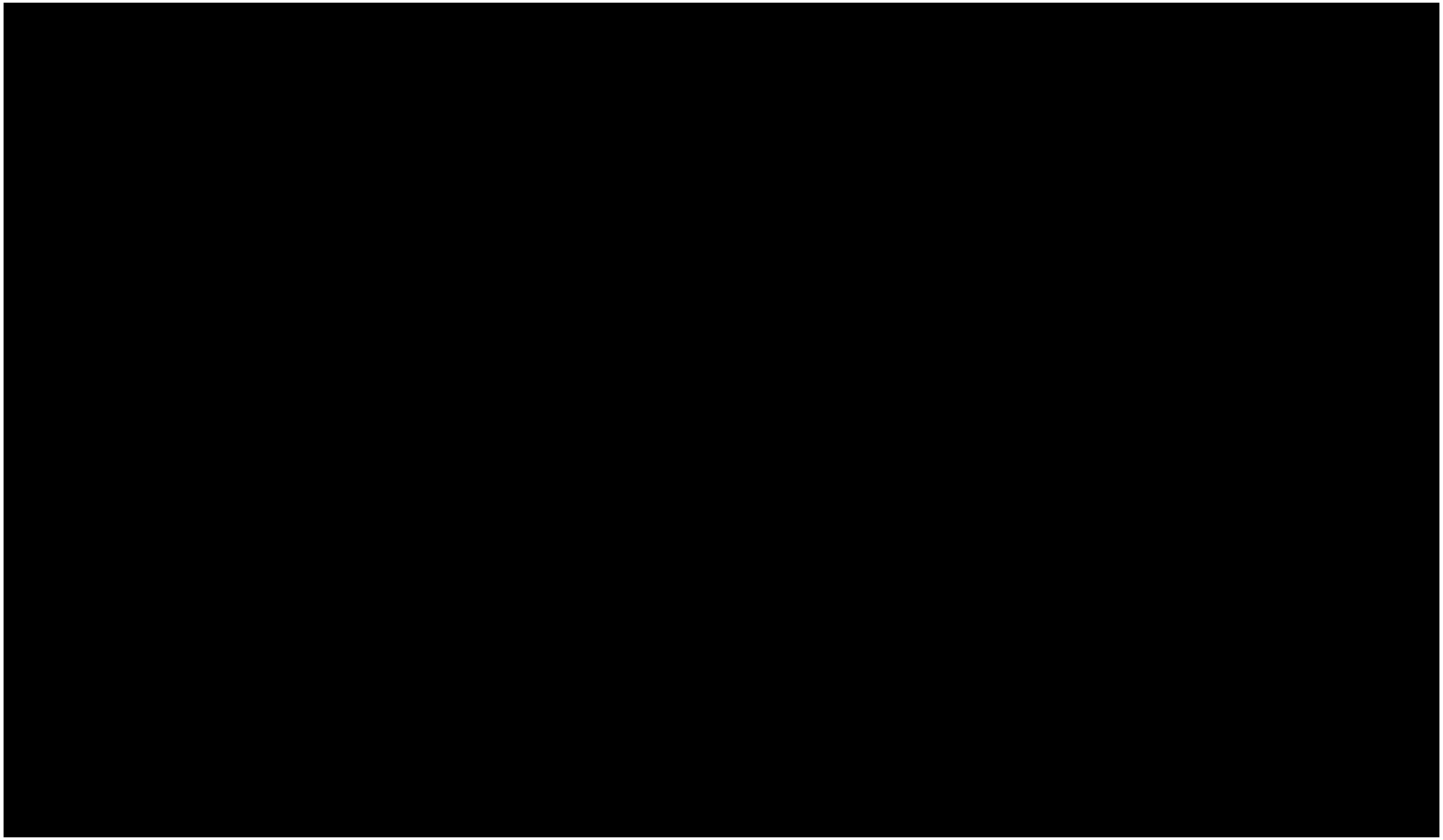
ascent → stabilization

descent→ destabilization

Necessary condition: $\frac{\partial\theta_e}{\partial z} < 0, \quad or \frac{\partial q_w}{\partial z} < 0$

Physical reason: demonstrate with a sketch

$$\frac{\partial \theta_e}{\partial z} < 0, \quad or \frac{\partial q_w}{\partial z} < 0$$



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Homework:

Notes: 2, 4, 6