HW 5 Ch. 5

ATS 541

10
15
14
10
49

2 : 15
3 : 14
4 : 5
6.4 : 10
44

1

$$f = \frac{e}{e_s} \qquad e_s = A e^{-B/T}$$

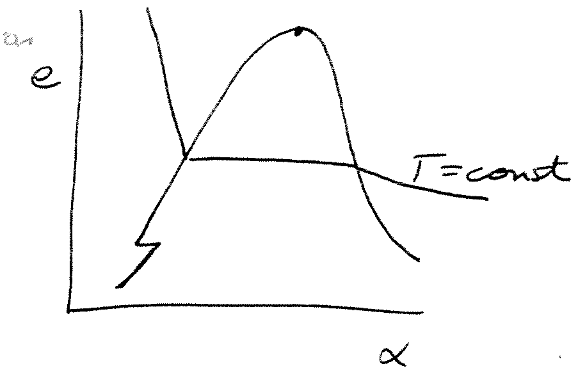
10 pts

$$= \frac{e}{A} e^{+B/T}$$

a) For $T = \text{const}$ — isothermal compression

$$\frac{\partial f}{\partial e} = \frac{1}{A} e^{+B/T} > 0$$

As $e \uparrow$, $f \uparrow$



b) Adiabatic expansion

$$e = c\left(\frac{T}{\theta}\right)^{1/\kappa}$$

θ = pot. temp.

c = const

Poisson's Eq.

$$f = \left(\frac{c\,\theta^{-1/\kappa}}{A}\right) T^{1/\kappa} e^{B/T} = c' T^{1/\kappa} e^{B/T} \quad \theta = \text{const}$$

$$\frac{\partial f}{\partial T} = c'\left(\frac{1}{\kappa} T^{1/\kappa - 1} - \frac{B}{T^2} T^{1/\kappa}\right) = \frac{c'}{\kappa} T^{1/\kappa - 1}\left(1 - \frac{\kappa B}{T}\right)$$

For $T < \kappa B = \frac{2}{7}(5420) = 1550\ K$, $1 - \frac{\kappa B}{T} < 0$

So $\frac{\partial f}{\partial T} < 0 \rightarrow$ expansion $\rightarrow$ Saturation

2. $L(T) = L_0 - (c_w - c_{pv})(T - T_0)$

15 pts

$$\frac{de_s}{dT} = \frac{e_s}{R_v T^2}\left[L_0 - (c_w - c_{pv})(T - T_0)\right]$$

$$= \left(\frac{a}{T^2} - \frac{b}{T}\right) e_s \qquad \text{where } a = \frac{L_0 + (c_w - c_{pv})}{R_v}$$

$$b = \frac{c_w - c_{pv}}{R_v}$$

$$\frac{de_s}{e_s} = \left(\frac{a}{T^2} - \frac{b}{T}\right) dT$$

$$\ln e_s = \ln e_{s0} + a\left(\frac{1}{T_0} - \frac{1}{T}\right) - b \ln\left(\frac{T}{T_0}\right)$$

$$e_s = e_{s0}\left(\frac{T}{T_0}\right)^{-b} \exp\left[a\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

or

$$e_s = e_{s0}\left(\frac{T}{T_0}\right)^{-\frac{c_w - c_{pv}}{R_v}} \exp\left[\frac{(L_0 + (c_w - c_{pv})T_0)}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

$e_{s0} = 611$ Pa , $T_0 = 0\,°C = 273.15$ K

T (°c)	Magnus (Pa) ~~(2.12)~~	$Ae^{-B/T}$ Eq. (5.5) (Pa)	Bolton (Pa)	Table 5.1 (Pa)
-30	50.85	52.05	51.04	51.06
-20	125.31	125.7	125.74	125.63
-10	286.20	283.8	286.8	286.57
0	611.0	603.8	611.2	611.21
10	1228.2	1217.7	1227.2	1227.94
20	2340.0	2341.1	2336.9	2338.54
30	4248.0	4310.7	4245.6	4245.20

3. $T = -5°C$ $p = 800$ mb $f = 0.65$

$\boxed{14 \text{ pts}}$

a) $\theta = T\left(\frac{100}{p}\right)^{\kappa(1-0.28 r_v)}$

Need r_v from part (b)

$= (268.15)\left(\frac{100}{80}\right)^{.286(1-0.28\cdot(.00213))}$

$= 285.77$ K

b) $r_v = r_{vs}(T = -5°C,\ p = 80\ kPa) \cdot f$

$= \frac{\varepsilon \cdot e_s(-5°C) \cdot f}{80\ kPa} = \frac{\overset{0.622}{\varepsilon}(421.84)(0.65)}{80 \times 10^3} = 2.13$ g/kg

c) $T_d = \frac{B}{\ln\left[\frac{A\varepsilon}{r_v p}\right]} = \frac{5.42 \times 10^3\ K}{\ln\left[\frac{(2.53 \times 10^8\ kPa)(0.622)}{(.00213)(80\ kPa)}\right]}$

$= 262.55$ K

d) $T_c = \frac{2840}{3.5 \ln T - \ln e - 4.805} + 55.$

$e = f \cdot e_s = (0.65)(421.84) = 274.20$ Pa

$= 2.74$ mb

$T_c = \frac{2840}{19.57 - 1.01 - 4.805} + 55.$

$= 261.4$ K

e) $\theta_e = \theta \exp\left(\frac{2675\, r_v}{T_c}\right)$

$= 285.77 \exp\left(\frac{2675 \cdot (.00213)}{261.4}\right)$

$= 292.07$

f) $T_v = T(1 + 0.61\, r_v) = 268.15\,(1 + (0.61)(.00213))$

$= 268.50\ K$

g) $\rho_m = \frac{P}{R_d T_v} = \frac{80{,}000\ Pa}{(287.05)(268.50)} = 1.04\ kg\ m^{-3}$

4. Given $T_d = 90° F = \frac{5}{9}(90-32) = 32.2 °C$

$p = 1000$ mb

$\boxed{\cancel{6} \text{ pts}}$

5 pts

find e, r_v

$$\begin{array}{r} 273.15 \\ \underline{32.2} \\ 305.35 \text{ K} \end{array}$$

$$e = e_s(T_d) = Ae^{-B/T_d}$$

$$\doteq 48.09 \text{ mb}$$

$\underline{49.6}$

$$r_v = \varepsilon \frac{e}{p-e} = 0.622 \frac{48.09}{1000-48.09} = 31.4 \text{ g kg}^{-1}$$

$\underline{32.013}$

$$\rho_v = \frac{e}{R_v T} = \frac{4960}{(461.5)(303.35)} = 0.0354 \text{ kg m}^{-3}$$

$$= \underline{35.4 \text{ g m}^{-3}}$$