ATS 541

Chap. 4

HW 4

1 : 15

2 : 10

3 : 10

4 : 5

5 : 10

50

C-3 [1] $m = 1$ g.

15 pts Processes considered:

i) Heat water from $T_1 = 0$ to $T_2 = 20°C$

ii) Evaporate all water ($m = 1$ g) @ $T_2 = 20°C$

a) Δu : Involves both latent, sensible heating and work

1 pt. From $dq = du + dw$ we can write

$$du = dq - p\,d\alpha$$

Note: dq (heating) $= m c_w dT$ (heating water)

dq (evap) $= m L_{lv}$ (evap. of water)

$$w = \int p\,d\alpha = p \int_{\alpha_l}^{\alpha_v} d\alpha = p(\alpha_v - \alpha_l) \simeq p\alpha_v$$

(assume work is involved only in the evaporation process) [$\alpha_v >> \alpha_l$]

Since $p\alpha_v = R_v T$, $\int p\,d\alpha \simeq p\alpha_v = R_v T = w$

Now,

$$\Delta u = m c_w \Delta T + m L_{lv} - m R_v T$$

($m w$ for 20°C)

$$= 10^{-3}\,\text{kg}\,[(4218\ \text{J kg}^{-1}\text{K}^{-1})(20\ \text{K}) + 2.45 \times 10^6\ \text{J kg}^{-1} - (461\ \text{J kg}^{-1}\text{K}^{-1})(293\ \text{K})]$$

$$= 2.40 \times 10^3\ \text{J}$$

1 cal = 4.184 J

Assume $c_w = 4218$ J kg^{-1} K^{-1} in above. (Iribarne & Godson)

Should use $c_w = 4190$ J kg^{-1} K^{-1} (Bolton 1980)

b) Δh

4pts

Since $dq = dh - \alpha \cancel{dp}^{0}$ (p = const)

$\Delta h = \int dh = \int dq$

$= m(c_w \Delta T + L_{lv}) = 10^{-3}\,kg\,[(4218\ J\,kg^{-1}K^{-1})(20K)$ (4190)

$+ 2.45 \times 10^{6}\ J\,k$

$= 2.53 \times 10^{3}\ J$

— — —

4pts

c) ΔS $\quad dS = \dfrac{dq}{T} = m c_w \int_{T_1}^{T_2} \dfrac{dT}{T} + \dfrac{m L_{vl}}{T_2}$

$= m\left(c_w \ln\left(\dfrac{T_2}{T_1}\right) + \dfrac{L_{vl}}{T_2}\right)$

$= 10^{-3}\,kg\left[(4218\ J\,kg^{-1}K^{-1}) \ln\left(\dfrac{293}{273}\right) + \dfrac{2.45 \times 10^{6}\ J\,kg^{-1}}{293\ K}\right]$

$= 8.83$ (8.67) $J\,K^{-1}$

2

$$\Delta S_{total} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4$$

10 pts

ΔS_1 : heat ice $T_1 = -10$ to $T_2 = 0$ °C

ΔS_2 : melt ice @ $T_2 = 0$ °C

ΔS_3 : heat water $T_2 = 0$ to $T_3 = 100$ °C

ΔS_4 : vaporize water @ $T_3 = 100$ °C.

$$\Delta S_i = mc_i \int_{T_0}^{T} \frac{dT}{T} = (.002\ kg)(2106\ J\ K^{-1}\ kg^{-1}) \ln \frac{293}{263}$$

$$= 0.157\ J\ K^{-1}$$

$$\Delta S_2 = m L_{sl} / T_2 = \frac{(.002\ kg)(3.34 \times 10^5\ J\ kg^{-1}\ K^{-1})}{273.15\ K}$$

$$= 2.45\ J\ K^{-1}$$

$$\Delta S_3 = m c_w \int_{T_2}^{T_3} d\ln T = (.002\ kg)(4180\ J\ kg^{-1}\ K^{-1}) * \ln\left(\frac{373}{273}\right)$$

$$= 2.61\ J\ kg^{-1}$$

$$\Delta S_4 = m L_{vl} / T_3 = \frac{(.002\ kg)(2.25 \times 10^6\ J\ kg^{-1}\ K^{-1})}{373.15\ K}$$ (for 100°C)

$$= 12.06\ J\ K^{-1}$$

$$\Delta S = \Sigma \Delta S_i = 17.28\ J\ K^{-1}.$$

3. $m = 200\ g$ $p = \text{const.}$

$\Delta S = 19.2\ J\,K^{-1}$

10 pts. $W = 1.61 \times 10^3\ J$

$$dq = du + \overset{(dw)}{p\,d\alpha} = c_p\,dT - \alpha\,dp \quad (dp = 0)$$

$$\Delta S = m\int \frac{dq}{T} = mc_p\int_{T_1}^{T_2}\frac{dT}{T} = mc_p \ln\left(\frac{T_2}{T_1}\right) = 19.2\ J\,K$$

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{19.2\ J\,K^{-1}}{0.2\ kg \cdot 1005.7\ J\,K^{-1}kg^{-1}} = .0955$$

$$\frac{T_2}{T_1} = 1.100$$

$$mw = W = m\int_{\alpha_1}^{\alpha_2} p\,d\alpha = m\,p\,(\alpha_2 - \alpha_1) \qquad p\alpha = RT,\ \alpha = \frac{RT}{p}$$

$$= m\,p\left(\frac{RT_2}{p} - \frac{RT_1}{p}\right)$$

$$1.61 \times 10^3\ J = m\,R\,(T_2 - T_1)$$

$$T_2 - T_1 = \frac{1.61 \times 10^3\ J}{m\,R_d} = \frac{1.61 \times 10^3\ J}{0.2\ kg \cdot 287.05\ J\,K^{-1}kg}$$

$$= 28.04\ K$$

Then $T_2 = 1.1\,T_1$

and $(1.1\,T_1 - T_1) = 0.1\,T_1 = 28.04\ K$

$$T_1 = 280.4$$

$$\therefore\ T_2 = T_1 + 28.04 = 308.44\ K$$

5 pts

4. Entropy conserved $\Rightarrow dS = 0 = \frac{dq}{T}$

This implies adiabatic, for which

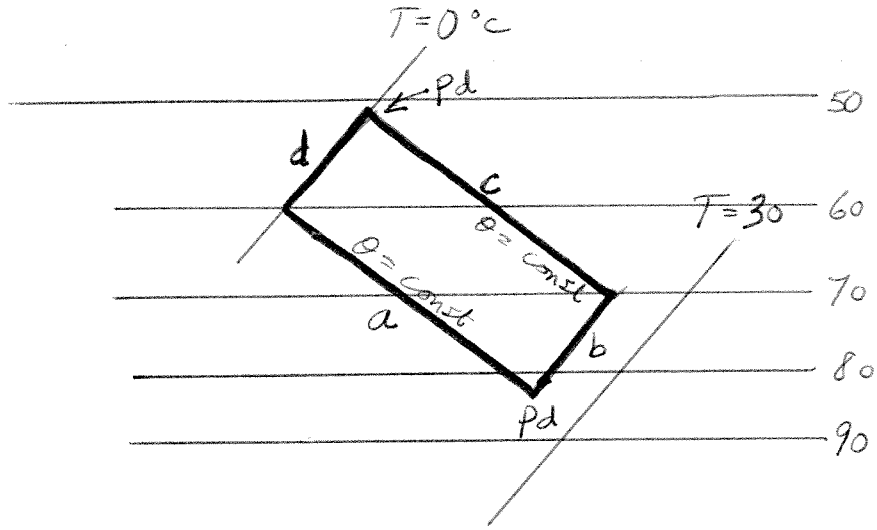
$$dq = 0 = du + dw$$

Since $dw > 0$ for an adiabatic expansion,

$du < 0$, so internal energy decreases.

5 min (ppts)

This Carnot cycle is traced out on the skew-T.



Section of a skew-T diagram

Two types of work involved

a) Work for an isothermal expansion (T=const)

$dW = p\,d\alpha$ $\qquad p\alpha = RT$, $\alpha = \frac{RT}{p}$

$$\boxed{dW = -RT\,d\ln p} \quad (1) \qquad d\alpha = -\frac{RT}{p^2}\,dp \quad (T=\text{const})$$

b) Work for an adiabatic change ($dq = 0$)

$$\boxed{p\,d\alpha = -c_v\,dT} \quad (2)$$

From (2), we see that work contributed by the adiabatic expansion/compression cancels since the temp. range is the same, but inverted.

$$W_a = -c_v \int_0^{25} dT \;; \quad W_c = -c_v \int_{25}^{0} dT$$

$$W_a + W_c = 0$$

The total work is the difference between the isothermal processes b & d.

$$W_b + W_d = -RT_b \int_{p_b}^{70} d\ln p \; -RT_d \int_{pd}^{6} d\ln p$$

(next page)

#5 (cont.)

We can determine p_b and p_d using

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\kappa} \quad \text{or} \quad \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{1/\kappa}$$

$$p = p_0 \left(\frac{T}{T_0}\right)^{1/\kappa}$$

So $$p_b = 60\left(\frac{298}{273}\right)^{1/\kappa} = 81.53 \text{ kPa}$$

$$p_d = 70\left(\frac{273}{298}\right)^{1/\kappa} = 51.51 \text{ kPa}$$

$\therefore$ $$w_{TOT} = w_b + w_d$$

$$= -R T_b \int_{81.53}^{70} d\ln p - R T_d \int_{51.51}^{60} d\ln p$$

$$= -R\left[(298\text{ K})\ln\left(\frac{70}{81.53}\right) + (273\text{ K})\ln\left(\frac{60}{51.51}\right)\right]$$

$$= (-287 \text{ J kg}^{-1}\text{K}^{-1})\,(-45.44 + 41.65)\text{ K}$$

$$= 1087.4 \text{ J kg}^{-1}$$