Chap. 7

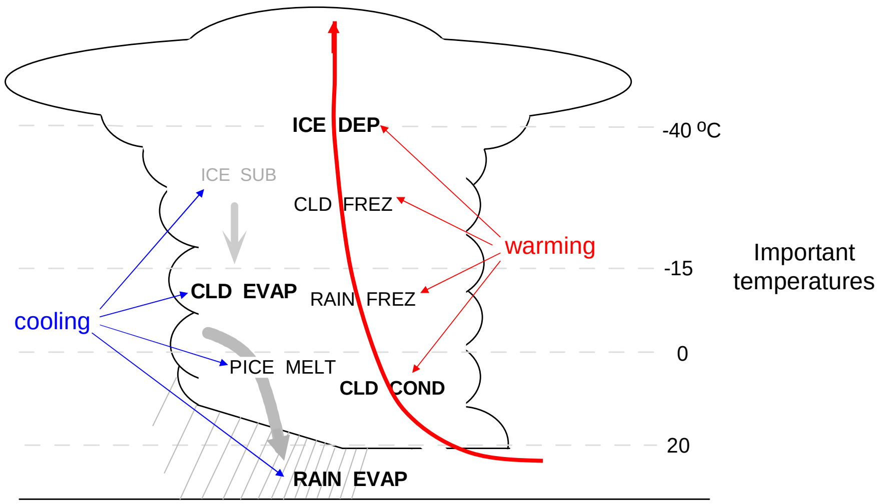
Common thermodynamic processes in the atmosphere

We will consider:

1. Real thermodynamic processes of importance in the atmosphere.
2. Underlying theme: latent heating or cooling
3. Link to cloud microphysical processes, (examined later in this course).

Example: Latent heating components in deep convection

Each latent heating component is associated with a phase change involving various particles and *microphysical processes*



7.2 Fog formation

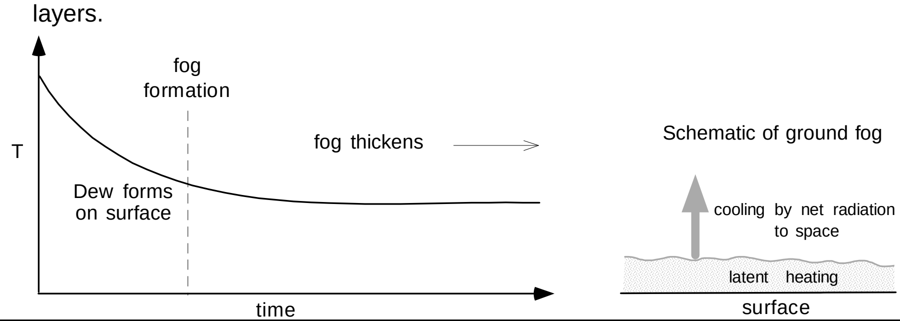
Several thermodynamic processes can produce fog:

a) local diabatic cooling via radiational cooling (e.g., radiation fog),

b) addition of water vapor (e.g., evaporation fog),

c) mixing of air parcels having different temperatures (e.g., advection fog).

For radiation fog, after saturation is attained, a negative heat flux (i.e., the upward conduction of cold air from the ground surface, from emission of long-wave radiation) results in condensation near the surface, i.e., production of fog layers.



Ground fog in the early morning

Consider three of six equations that were used by Girard and Jean-Pierre (2001) to form a simple cloud/fog model:

Local change | diffusion | Sources/sinks

Water vapor mixing ratio

$$\frac{\partial q}{\partial t} = K\frac{\partial^2 q}{\partial z^2} + S_{I_{\text{depo}}} + S_{W_{\text{cond}}} + S_{A_{\text{cond}}} + S_{A_{\text{depo}}},$$

Temperature

$$\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial z^2} + \frac{Q_{\text{rad}}}{C_p} + \frac{Q_{\text{cond}}}{C_p} + \frac{Q_{\text{depo}}}{C_p},$$

Physical basis?

Saturation ratio

$$\frac{d\delta}{dt} = -\left(\frac{L_v}{R_v T^2}\right)\frac{\partial T}{\partial t} - \left(\frac{p}{\varepsilon e_s}\right) \times [S_{W_{\text{cond}}} + S_{I_{\text{depo}}} + S_{A_{\text{cond}}} + S_{A_{\text{depo}}}],$$

What familiar equation do you see here?

Conservation equations for water vapor mixing ratio (q), temperature (T) and saturation (δ, think of this in terms of relative humidity)

Now, back to the radiation fog problem:

From the isobaric form of the First Law (radiational cooling – net emission of long-wave radiation – is related to dq)

$dq_{rad} = dh = c_p dT + L_{vl} dr_{vs}$ (dp=0)
(7.1)

Using the relation between r_{vs} and e_s

$r_{vs} = \varepsilon e_s/p$ -> $dr_{vs} = \varepsilon de_s/p$ (dp=0)

together with the Clausius-Clapeyron eq.

$de_s/e_s = L_{vl} dT/(R_v T^2)$

we can write

$dr_{vs} = [\varepsilon/p]de_s = [\varepsilon L_{vl} e_s / pR_v T^2]dT$ (7.2)

Substitution of Eq. (7.2) into Eq (7.1) yields

$$dq = c_p dT + \left[\frac{\varepsilon L_{vl}^2 e_s}{R_v p}\right]\frac{dT}{T^2}$$

$$= \left[c_p + \frac{\varepsilon L_{vl}^2 e_s}{pR_v T^2}\right]dT \qquad (7.3)$$

Or, using the alternate version of (7.2) we can write

$$dq = \left[\frac{c_p R_v T^2}{L_{vl} e_s} + \frac{\varepsilon L_{vl}}{p}\right]de_s \qquad (7.4)$$

Eq (7.3) can be used to compute the ΔT if a corresponding Δq (IR radiational cooling) can be measured. The decrease in saturation vapor pressure (e_s) can be obtained from (7.4).

The mass of water vapor per unit volume can be obtained from the equation of state for water vapor:

$$\rho_v = e_s/R_v T$$

Differentiation of this equation yields

$$d\rho_v = \frac{de_s}{R_v T} - \left(\frac{e_s}{R_v T^2}\right) dT \approx \frac{de_s}{R_v T} \qquad (7.5)$$

(This approximation is accurate to within ~5%; which can be shown with the C-C eq. or even with a more simplistic scale analysis.)

As cooling produces condensation in the fog, the differential amount of condensate (mass per unit volume) can be found using (7.5) with the C-C eq. as

$$dM = \frac{-de_s}{R_v T} = -\left(\frac{L_{vl} e_s}{R_v^2 T^3}\right) dT \qquad (7.6)$$

Example:

Using (7.6), find the cooling required to form a fog liquid water content of 1 g m^{-3} (a vary large value) if the air is saturated at 10 °C.

From (7.6) we have

$\Delta T = -\Delta M[R_v^2 T^3 / L_{vl} e_s(T)]$

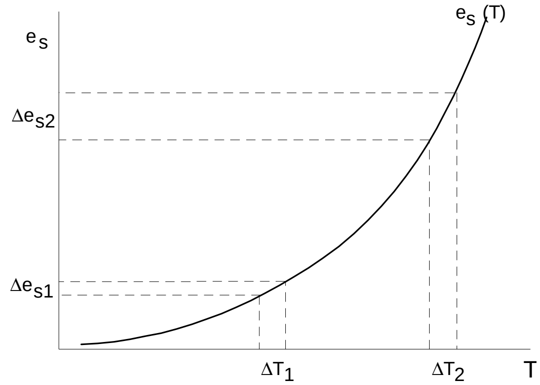
$= -10^{-3}$ kg m^{-3} $[(461 \text{ J K}^{-1} \text{ kg}^{-1})^2 (283 \text{ K})^3 / (2.5\text{x}10^6 \text{ J kg}^{-1})(1227 \text{ Pa})$

$= -1.57$ K

Using (7.5) the ΔM relates to Δe_s by $-\Delta e_s = R_v T \Delta r_c$.

For ΔM=1 g m^{-3}, Δe_s is 1.3 to 1.4 mb over a broad range of T.

Examination of a plot of e_s vs. T (Fig. 7.3), shows that the ΔT required for a given production of condensate (ΔM) increases as T decreases. ***This suggests that thick fogs are less likely at low T.***



Some considerations of the previous statement:

Is this consistent with your observation?
Fog tends to be more frequent during the cold months. Why? (What "forcing" is required for fog formation? A good answer requires some knowledge on radiative transfer.)

Why are ground fogs more prevalent in September than in June (at least for the Huntsville region)?

7.3 Effects of freezing in a cloud system (pp. 124-126, I&G)

From the First Law, freezing of supercooled water within a cloud will produce a temperature increase (assuming all supercooled liquid water is instantaneously converted to ice – and this is a good assumption) according to

$$dq = -L_{il} dr_c = c_p dT \qquad \text{(isobaric process)} \qquad (7.8)$$

This is only the first order approximation and does not account for all the physics. Rather, three processes (enthalpy components) should be considered:

1) Latent heat of freezing:

$$\Delta H_1 = - L_{li} dr_c = L_{li} dr_{ice} \qquad (dr_c = -dr_{ice}) \qquad (7.9a)$$

2) Depostion of water vapor on the newly formed ice particle (this occurs because of the difference in $e_{sv}(T)$ and $e_{si}(T)$ – see Table 5.1):

$$\Delta H_2 = -L_{vi}[(r_{vs}(T)-r_{vi}(T)] \qquad (7.9b)$$

3) Absorption of the latent heat by dry air, water vapor and the newly-formed ice condensate:

$$\Delta H_3 = [\underbrace{c_{pd}}_{\text{dry air}} + \underbrace{r_{si}(T')c_{pv}}_{\text{water vapor}} + \underbrace{r_{ice}c_i}_{\text{ice condensate}}] \cdot (T' - T) \qquad (7.9c)$$

The saturation vapor mixing ratios in (7.9b) be related to vapor pressure, utilizing the often-used relation

$$r_{vs} \cong \frac{\varepsilon e_{vs}(T)}{p}, \qquad r_{vi} \cong \frac{\varepsilon e_{si}(T')}{p}, \tag{7.10}$$

along with the Clausius-Clapeyron equation ($d\ln e_s = (L_{vl}/R_vT_2)dT$), to express $e_{si}(T')$ in terms of the initial temperature (T) and the temperature difference (T′-T), assuming that (T′-T) is small enough to be treated as a differential, e.g.,

$$e_{si}(T') = e_{si}(T) + \frac{L_{vi}e_{si}(T)}{R_vT^2}(T'-T) \tag{7.11}$$

.

Combining 7.10 and 7.11 into 7.9b:

$$\Delta H_2 = -\frac{\varepsilon L_{vi}}{p}\left[e_{vs}(T) - e_{si}(T) - \frac{L_{vi}(T)e_{si}(T)}{R_vT^2}(T'-T)\right]$$

$$= -r_{vs}(T)L_{vi}\left[1 - \frac{e_{si}(T)}{e_{vs}(T)}\right] + \frac{r_{si}(T)L_{vi}{}^2}{R_vT^2}(T'-T) \tag{7.12}$$

From heat balance considerations (latent heating is balanced by an increase in H) we can write

$$\Delta H_1 + \Delta H_2 + \Delta H_3 = 0$$

and then solve for $\Delta T = (T'-T)$ to get

$$\Delta T = \frac{L_{vi}r_{ice} - L_{vi}r_{vs}\left(1 - \frac{e_{si}}{e_{vs}}\right)}{c_p + \frac{r_{si}L_{vi}{}^2}{R_vT^2}} \tag{7.13}$$

The c_p term includes $r_{si}(T')$ which is not known, but can be obtained if desired by numerical solution (successive approximations). The contributions from this term are small since $r_{si}(T')\cdot c_{pv} << c_{pd}$. When r_{vs} and r_{si} are small, (7.13) can be well approximated by

$$\Delta T = r_{ice}L_{vi}/c_p. \tag{7.14}$$

This latent can be an important source of local heating in convective cloud systems since the heating proceeds relatively rapidly over a relatively shallow depth. Such rapid conversion of supercooled water to ice is termed *glaciation*. For example, if 5 g kg^{-1} of supercooled water is converted to ice, then from (7.14), $\Delta T = 1.7$ K, which in general is a significant increase in bouyancy.

This concept was vigorously pursued during the 1970's over south Florida, where cloud systems were studied and seeded in order to evaluate the ***dynamic response**** (and subsequent upscale cloud system growth) of rapid glaciation in the mixed phase region of clouds. (The mixed phase region is defined as the region, typically between temperatures of -20 and 0 °C, where ice and supercooled water coexist, within updraft regions of cloud systems).

* In this dynamic response, it was hypothesized that the net latent heating would accelerate updrafts, thereby reducing pressure near the surface, which in turn would increase mass convergence in the boundary layer.

7.4.1 Melting in stratiform precipitation (see also Rogers and Yau, pp. 197-203)

Stratiform precipitation is a common. One important characteristic of melting is that it proceeds relatively rapidly over a relatively shallow depth, typically in the range 100-500 m. Local cooling rates within melting regions can be appreciable. Before proceeding with cooling by melting, we will first examine the factors that govern the local rate of cooling within mesoscale precipitation systems. Starting with the First Law, we have

$$dq = c_p dT - \alpha dp.$$

Dividing both sides by the time differential dt and solving for dT/dt (we are after rates of cooling here) yields

$$c_p \frac{dT}{dt} = \alpha \frac{dp}{dt} + \frac{dq}{dt} \quad (7.15)$$

We now use the equation of state to substitute for α [α=RT/p], and use a definition from atmospheric dynamics, $\omega \equiv dp/dt$. Eq. (7.15) then becomes

$$\frac{dT}{dt} = \frac{RT}{c_p p}\omega + \frac{1}{c_p}\left(\frac{dq}{dt}\right)_{dia} \quad (7.16)$$

rate of diabatic heating (cooling by melting)

To find the local change we decompose the total derivative into the local and advective changes:

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + V\frac{\partial T}{\partial s} + \omega\frac{\partial T}{\partial p}$$

where V is the horizontal wind and s is in the (horizontal) direction of the flow. Substituting this into (7.16) and solving for local term $\partial T/\partial t$ gives

$$\frac{\partial T}{\partial t} = \underbrace{-V\frac{\partial T}{\partial s}}_{\textbf{i}} + \underbrace{\omega\left(\frac{RT}{c_p p} - \frac{\partial T}{\partial p}\right)}_{\textbf{ii}} + \underbrace{\frac{1}{c_p}\left(\frac{dq}{dt}\right)_{dia}}_{\textbf{iii}} \qquad (7.17)$$

Local changes in T are accomplished by

(i) horizontal temperature advection,
(ii) vertical motion and
(iii) diabatic heating.

In actual precipitation systems, there are instances where effects from term (iii) dominated by terms (i) and/or (ii).

For melting (term iii), the temperature change can be found from a form similar to that of Eq (7.14),

$r_i\, dq = r_i L_{il} = r_i c_p\, dT$,

where r_i is the mixing ratio of ice precipitation.

For melting, $(dq/dt)_{dia} = L_{il}$ * (rate of precipitation), and we can write

$\Delta m_i L_{il} = \rho_d c_p H A \Delta T$

where m_i is the mass of ice, ρ_d the density of air, H the melting depth and A the unit area. [Also used $\rho_d = m_d/V = m_d/HA$). Converting m_i to a precipitation rate R (mm hr^{-1}), and dividing through by Δt yields

$(\Delta T/\Delta t) = RL_{il} / c_p\rho_d H$ (average over depth H and Δt) (7.18)

For a precipitation rate of 2 mm hr^{-1} (a modest rate in stratiform precipitation) and H=400 m, a cooling rate of 1.6 K/hr is produced.

The depth H over which the melting occurs is a function of ice particle size, precipitation rate R and relative humidity.

One consequence of this prolonged diabatic heating is the formation of an isothermal layer, increased static stability within this layer, and often a decoupling of the atmosphere above and below the melting layer.

[Insert a conceptual picture here]

[Also think about the initial value problem.]

7.4.3 Thermodynamics within thunderstorm downdrafts

Consider an air parcel moving downward within a precipitation environment in the lower levels of a precipitating cumulonimbus cloud system.

Diabatic cooling sources include evaporation and melting.

Changes in θ (following a parcel, so we consider the total derivative) are accomplished by evaporation of water, melting, and sublimation of ice:

$$\underbrace{\frac{d\theta}{dt}}_{\text{total change}} = \frac{\theta_o}{c_p T_o}\left[L_{vl}\left(\underbrace{VD_{rv}}_{\text{rain}} + \underbrace{VD_{cv}}_{\text{cloud}}\right) + L_{vi}\underbrace{VD_{gv}}_{\text{sublimation of ice particles}} + L_{il}\underbrace{ML_{gr}}_{\text{melting of ice particles}}\right] \qquad (7.19)$$

(evaporation of: rain, cloud)

θ_o, T_o are initial parcel values of θ and T,
VD_{rv} is the rate of evaporation of rain,
VD_{cv} is rate of evaporation of cloud,
VD_{gv} is rate of sublimation of ice,
ML_{gr} is rate melting of ice