Chap. 6

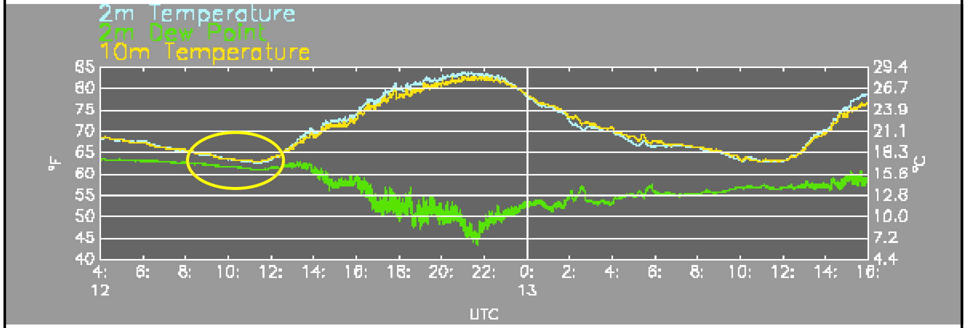
ATMOSPHERIC THERMODYNAMIC PROCESSES

[see also Petty, Section 7.5-7.10, pp. 188-237]

Objectives:

1. Develop other important thermodynamic variables and applications of the fundamental relations that we have considered to this point.
2. Use and applications the skew-T diagram to examine atmospheric processes.
3. Examine some important atmospheric thermodynamic processes. In particular, we will explore the behavior of water vapor and its effects on atmospheric processes.

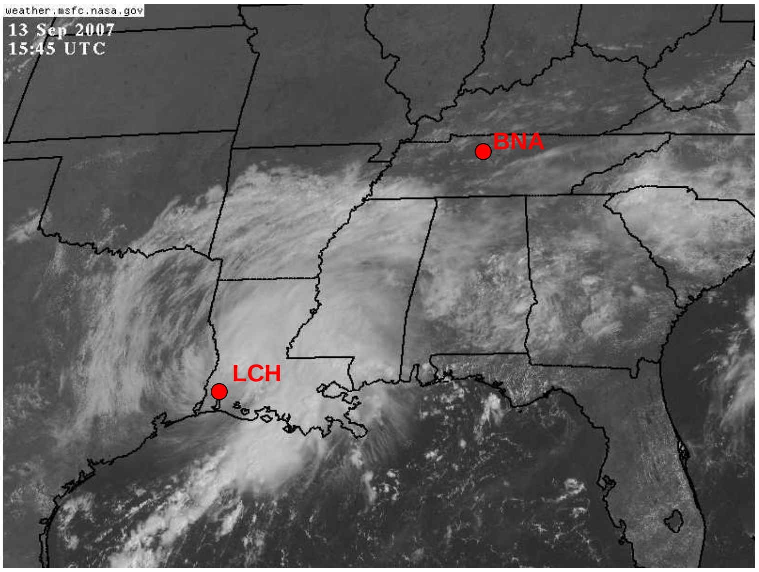
Saturation is common in the atmosphere



Near the surface on clear, calm nights

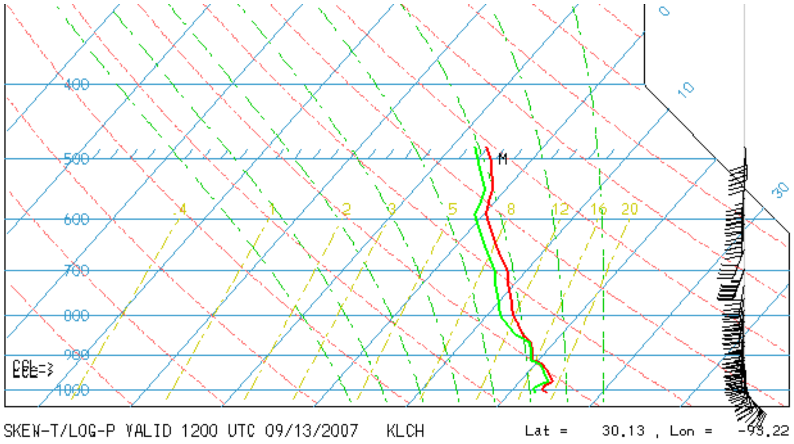
http://vortex.nsstc.uah.edu/mips/data/current/surface/

Saturation is common in the atmosphere

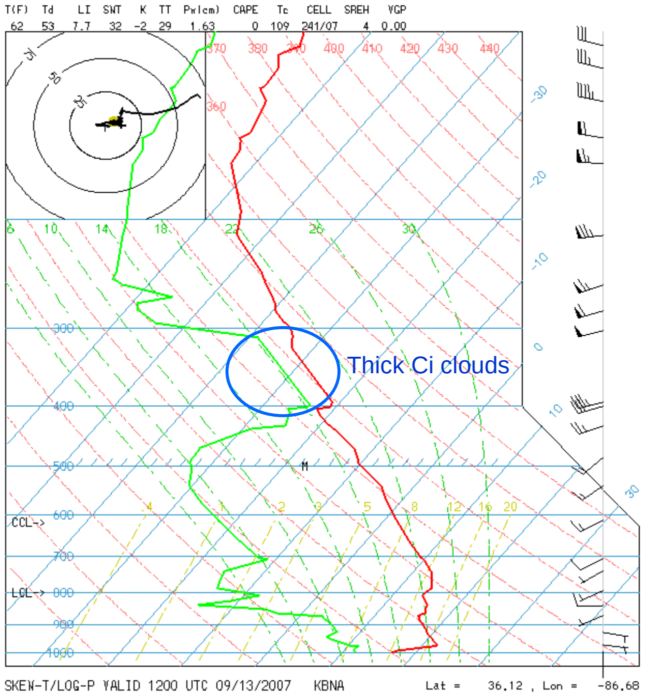


Formation, evolution, and movement of clouds

Saturation is common in the atmosphere



Depiction of saturation in a skew-T, ln p sounding (1200 UTC, 13 Sept 2007, Lake Charles, LA)



Thick Ci clouds

6.1.1 Some processes that define additional thermodynamic variables

There are *four natural processes by which saturation can be attained in the atmosphere*. These are:

- ***isobaric cooling*** ($dq \neq 0$, r_v=const), e.g., by radiative cooling (diabatic cooling, $dq < 0$), in which the temperature T approaches the dew point temperature T_d;
- ***evaporational cooling*** ($dq \neq 0$, $rv \neq$const) in which a decrease in T and an increase in T_d result in the wet-bulb temperature T_w, (at which point the air is saturated);
- ***adiabatic cooling*** ($dq=0$, r_v=const) in which saturation is produced at the saturation point temperature[1] T_{sp} by adiabatic expansion;
- ***mixing* of two air masses** – in this case saturation can be analyzed from a “saturation point” mixing analysis.

[1] The saturation point temperature is also known as the temperature of the lifting condensation level (T_{lcl}) or the isentropic condensation temperature (T_c).

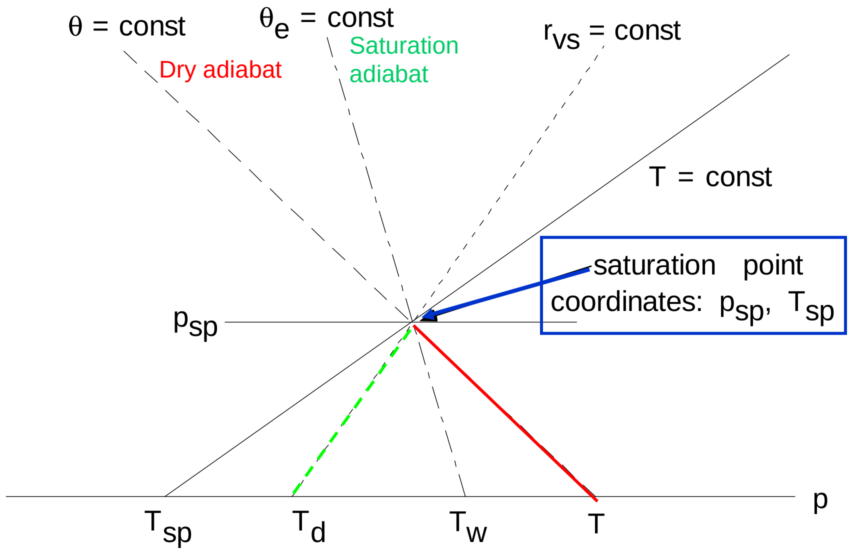


Figure 6.2. Illustration of processes by which saturation may be achieved in the atmosphere. This skew-T diagram also illustrates the graphical method to determine T_d, T_w and T_{sp}. Illustration of Normand's rule. **This is an important figure!!!**

Upper air observations
http://www.vaisala.com/businessareas/measurementsystems/soundings/products/radioso

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a) *Isobaric cooling and the dew point temperature, Td.*

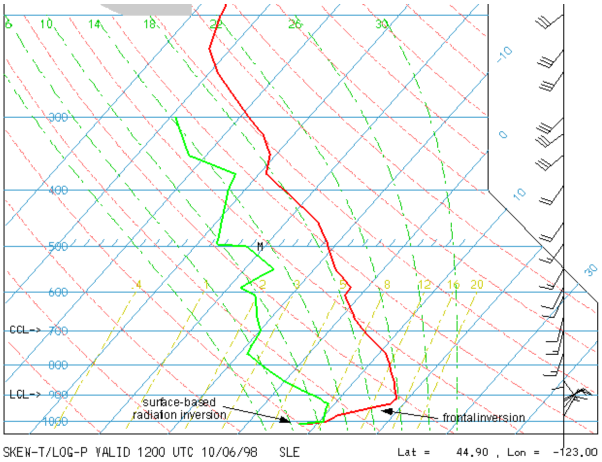
isobaric process

(radiational) cooling occurs in the presence of constant water vapor (e=const or r_v=const).

Under clear sky conditions the radiational cooling frequently reduces the surface temperature to the dewpoint temperature.

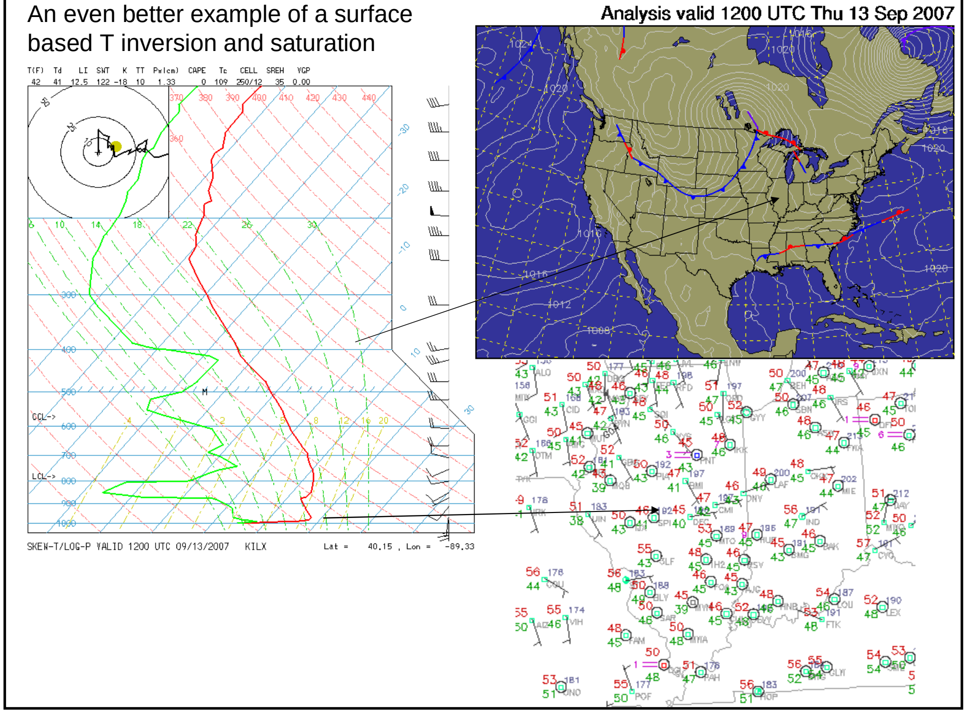
Cooling is greatest at the surface, as illustrated in Fig. 6.1, a 1200 UTC sounding from Salem, Oregon.

T and T_d are nearly superimposed (i.e., the air is saturated) and fog was reported in the region.



This sounding was obtained from the web site
http://www.rap.ucar.edu/weather/upper.html

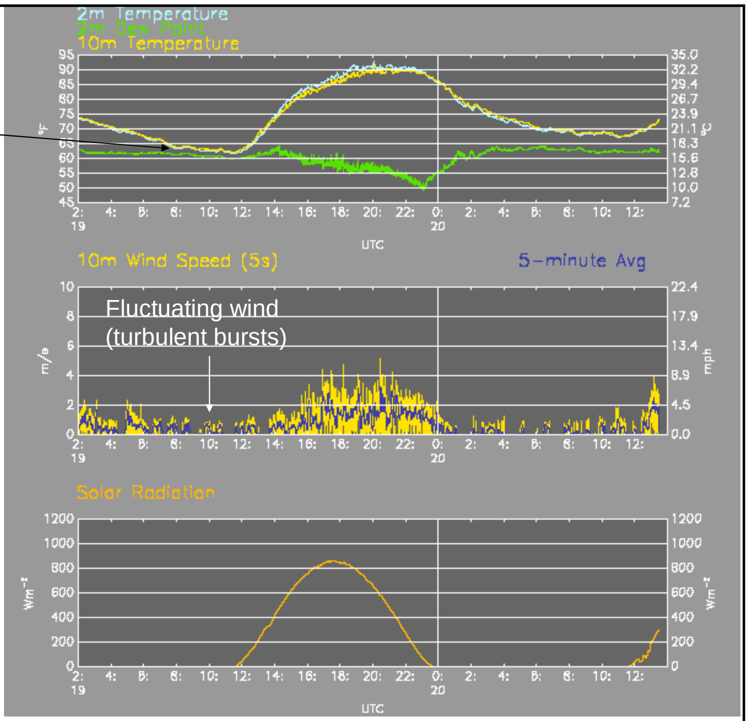
An even better example of a surface based T inversion and saturation



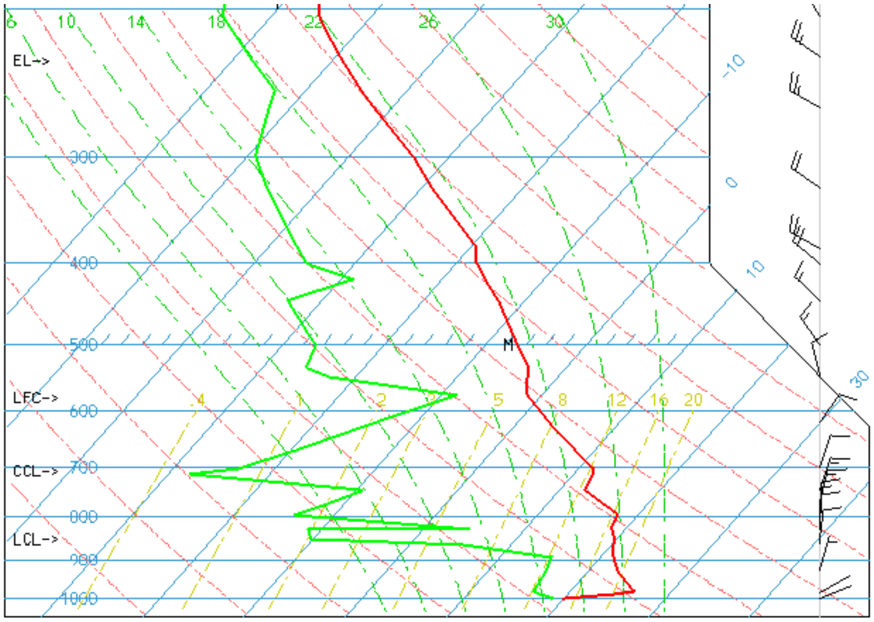
$T \rightarrow T_d$
(RH → 100%)

Low winds in the NBL
+ clear skies
+ dry air above 1 km
= maximum surface cooling

Relation between Δq_{rad} and ΔT



Fluctuating wind (turbulent bursts)



SKEW-T/LOG-P VALID 1200 UTC 09/20/2005 KBMX Lat = 33.17 , Lon = -86.77

Further illustration: For the next two weeks, refer to T, T_d time series over night from http://vortex.nsstc.uah.edu/mips/data/current/surface/.

Derivation of T_d:

dp=0, dq≠0 and dh=dq.

Physical process: As isobaric cooling proceeds with no change in the absolute moisture content, a temperature is reached in which the air just becomes saturated ($T=T_d$, or $e = e_s$).

Consider the relationship between T_d and the relative humidity f. We can write

$r_v = r_{vs}\,(p,T_d)$ (definition)

and use an expression for $e_s(T)$ and the approximate relation $r_{vs}=\varepsilon e_s/p$.

We begin with the integrated approximation of the C-C equation:

$$e_s = Ae^{-B/T} \tag{6.1}$$

Take the natural log of each side (i.e., $\ln e_s = \ln A - B/T$), utilize the approximate formula

$e_s=pr_{vs}/\varepsilon$,

and then solve for T (which is T_d in this case).

The approximate analytical expression for T_d in terms of r_v and p can then be expressed as

$$T_d = T_d(r_{vs}, p) = \frac{B}{\ln\left(\frac{A\varepsilon}{r_v p}\right)} \qquad (6.2)$$

This relation explicitly shows that T_d is a function of r_v and p. Given values of r_v and p, one can graphically determine T_d on a skew-T as shown in Fig. 6.2 below. As an extension of this problem, we will consider fog formation in Section 6.4.1.

The accuracy of this relation is dependent on the accuracy of $e_s = Ae^{-B/T}$ [accurate to about 1% in the -20 to +20 C interval]

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

From Petty.

Relationship between T_d and relative humidity f.

Determine how relative changes in T_d are related to relative changes in f. Again, we utilize the formula

$$e \cong \frac{pr_v}{\varepsilon}$$

and take the log differential to get

$$\mathrm{dlne = dlnp + dlnr_v}. \qquad (6.3)$$

Combine (6.3) with the Clausius-Clapeyron equation, written in differential form

$$\frac{\mathrm{d\ln e}}{\mathrm{dT_d}} = \frac{\mathrm{L_{vl}}}{\mathrm{R_v T_d}^2} \qquad \text{(here e = } e_s \text{ since T=} T_d\text{)} \qquad (6.4)$$

to obtain, after some rearranging, the following:

$$\mathrm{dT_d} = \frac{\mathrm{R_v T_d}^2}{\mathrm{L_{vl}}}(\mathrm{d\ln p + d\ln r_v})$$

,

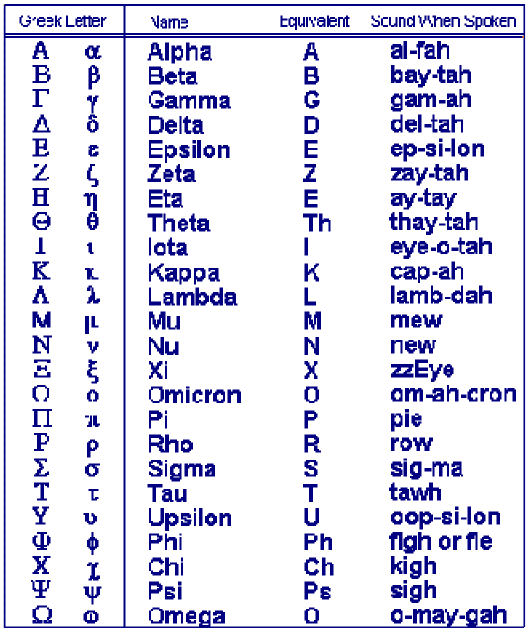
Depends on relative variations in p and r_v

Dividing both sides by T_d yields

$$d\ln T_d = \frac{dT_d}{T_d} = \frac{R_v T_d}{L_{vl}}(d\ln p + d\ln r_v) \approx 5x10^{-2}(d\ln p + d\ln r_v)$$

where the latter approximate equality is obtained from the term $[R_vT_d/L_{vl}]$ by assuming T_d=270 K, L_{vl}=2.5x10^6 J kg^{-1}, and R_v= 461 J kg^{-1} K^{-1} . This result indicates that the relative increase in T_d (here, relative refers to the ratio dT_d/T_d, or an incremental change relative to the total value) is about 5% the sum of the relative increases in p and r_v. We now integrate the C-C eq. (6.4) above to get

$$\ln\left(\frac{\mathrm{e_s}}{\mathrm{e}}\right) = -\ln \mathrm{f} = \frac{1}{\mathrm{R_v}}\int_{\mathrm{T_d}}^{\mathrm{T}} \frac{\mathrm{L_{vl}}}{\mathrm{T}^2}\mathrm{dT} \cong \frac{\mathrm{L_{vl}}}{\mathrm{R_v}}\frac{\mathrm{T - T_d}}{\mathrm{TT_d}}.$$



Greek Letter		Name	Equivalent	Sound When Spoken
Α	α	Alpha	A	al-fah
Β	β	Beta	B	bay-tah
Γ	γ	Gamma	G	gam-ah
Δ	δ	Delta	D	del-tah
Ε	ε	Epsilon	E	ep-si-lon
Ζ	ζ	Zeta	Z	zay-tah
Η	η	Eta	E	ay-tay
Θ	θ	Theta	Th	thay-tah
Ι	ι	Iota	I	eye-o-tah
Κ	κ	Kappa	K	cap-ah
Λ	λ	Lambda	L	lamb-dah
Μ	μ	Mu	M	mew
Ν	ν	Nu	N	new
Ξ	ξ	Xi	X	zzEye
Ο	ο	Omicron	O	om-ah-cron
Π	π	Pi	P	pie
Ρ	ρ	Rho	R	row
Σ	σ	Sigma	S	sig-ma
Τ	τ	Tau	T	tawh
Υ	υ	Upsilon	U	oop-si-lon
Φ	φ	Phi	Ph	figh or fie
Χ	χ	Chi	Ch	kigh
Ψ	ψ	Psi	Ps	sigh
Ω	ω	Omega	O	o-may-gah

We then solve for so-called *dewpoint depression* $(T\text{-}T_d)$, use decimal logarithms [using the definition that $\log_{10} \xi = \ln \xi / \ln 10 = 0.43429 \ln \xi$], and substitute for constants to get

$$(T\text{-}T_d) = 4.25\text{x}10^{-4}\ T{\cdot}T_d(-\log10f)$$

For $T{\cdot}T_d = 290^2$ (i.e., assuming $T = 290$ K and $T_d = 290$ K) we have

$$T\text{-}T_d \approx 35(-\log10f) \qquad (6.5)$$

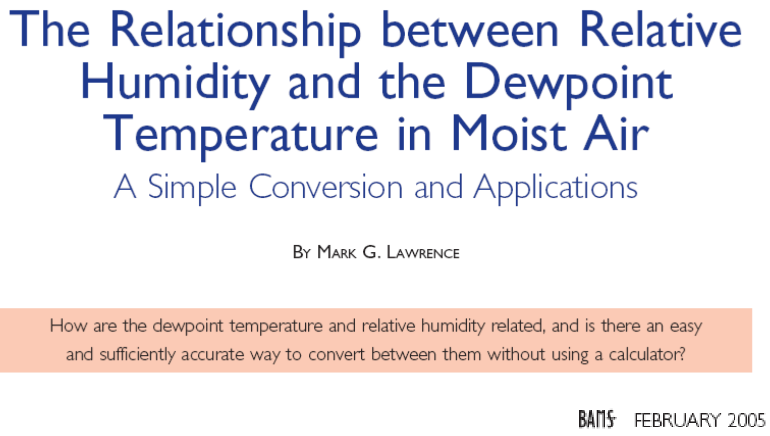
Then for f=0.8, $(T\text{-}T_d) \cong 3.5$ °C = 6.3 °F. Thus, a change of $(T\text{-}T_d)$ every 1 °F translates to a change in f of about 3.2%.

This verifies my general rule of thumb that [at least] for f > 0.8, the dewpoint depression, $T\text{-}T_d$, is 1 °F for every 3% change in f, for f < 100%.

For example, if T=75 °F and f=0.88, then $T_d \cong 71$ °F.

Observational question: *What is the range of Td in the atmosphere*? What is the upper limit of Td, and where would this most likely occur?

A reading assignment:



The Relationship between Relative Humidity and the Dewpoint Temperature in Moist Air

A Simple Conversion and Applications

By Mark G. Lawrence

How are the dewpoint temperature and relative humidity related, and is there an easy and sufficiently accurate way to convert between them without using a calculator?

BAMS FEBRUARY 2005

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b) ***Isobaric wet-bulb temperature** (T_{iw})*

We will consider the isobaric wet-bulb temperature T_{iw} here – there is also an adiabatic wet-bulb temperature, T_{aw}. The wet-bulb temperature is achieved via the process of *evaporation*. Practical examples of T_{iw} are evaporation of rain and the evaporation of the wet bulb wick on the sling psychrometer, a device which measures the dry and wet-bulb temperatures. While the process is isobaric (ideally), the parcel gains r_v at the expense of a decrease in T. Assuming that a parcel of unit mass (1 kg) contains r_v kg kg^{-1} of water vapor, we can write from the First Law (p=const)

$$dq = c_{pd}(1+0.887r_v)dT = c_{pm}dT \qquad [c_p=c_{pd}]$$

The heat loss from evaporation (including a mass r_v of water vapor) is

$$(1+r_v)dq = -L_{lv}dr_v$$

Equating the two expressions above gives

$$c_{pd}dT = -L_{lv}dr_v[1/(1+r_v)][1/(1+0.9r_v)] \cong -L_{lv}dr_v(1-1.9r_v)$$

$$c_pdT \cong -L_{lv}dr_v \quad \text{(within \textasciitilde 2\%, since } r_v \sim 0.01) \qquad (6.7)$$

6.1.1 Some processes that define additional thermodynamic variables

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[1] The saturation point temperature is also known as the temperature of the lifting condensation level (T_{lcl}) or the isentropic condensation temperature (T_c).

Assume that L_{lv} and c_p are constant (which is a good assumption since the temperature reduction $\Delta T = T - T_{iw}$ associated with evaporation is typically <10 K)

Integrate the above to get the wet-bulb depression ($T - T_{iw}$)

$$T - T_{iw} = (L_{vl}/c_p)(r_{vs}(T_{iw},p) - r_v).$$

Use the Clausius-Clapeyron formula (5.5) [$e_s(T) = e^{-B/T}$] to get an iterative formula for T_{iw}:

$$T_{iw} = T - (L_{lv}/c_p)[(\varepsilon/p)Ae^{-B/T}{}_{iw} - r_v], \qquad (6.8)$$

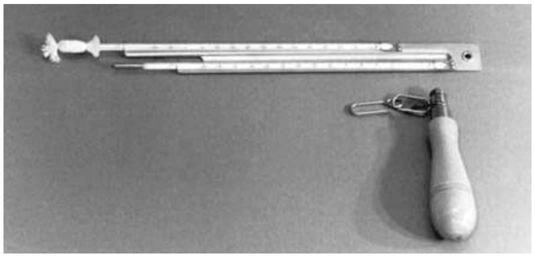
where T and r_v are the initial parcel values.

On the psychometric equation (Bohren and Albrecht, pp 282-284):

This equation provides a relation between vapor pressure (e) and the wet bulb depression, ($T-T_{iw}$). T is termed the dry-bulb temperature, and T_{iw} the wet bulb temperature. Both can be measured with a sling psychrometer (or an Assman aspirated psychrometer, see Fig. 7.19); these measurements are used to determined the vapor pressure e. (In the “old days” tables were used.)

$$e = e_s(T_{iw}) - \frac{pc_{pd}}{\varepsilon L_{vl}}(T - T_{iw})$$

Derivation? Same starting point: $c_p dT \cong -L_{lv} dr_v$



Sling psychrometer (hand-held)

http://asd-www.larc.nasa.gov/SCOOL/psychrometer.html
http://www.novalynx.com/225-520.html
http://www.climatronics.com/Applications/Sensors-and-Components/index.php

Handheld Relative Humidity and Temperature Meter

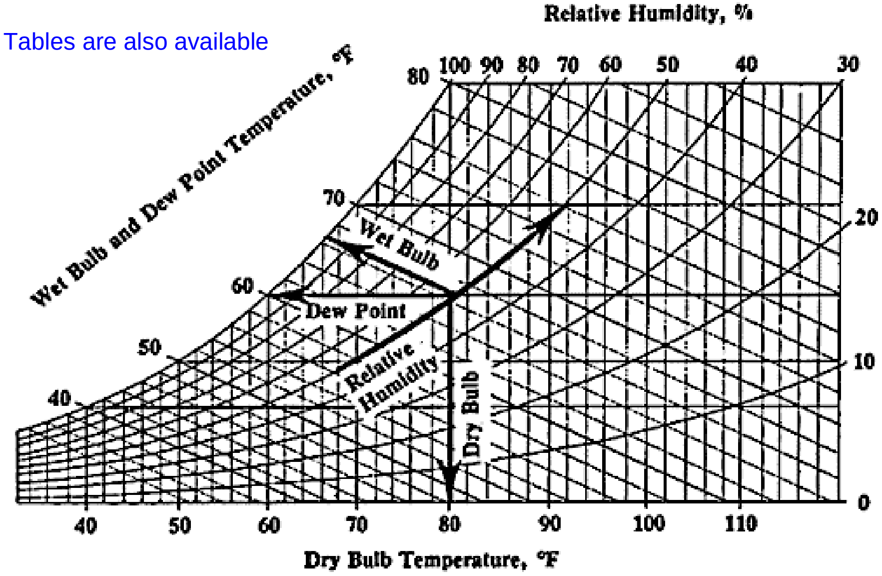
http://www.omega.com/toc_asp/subsectionSC.asp?subsection=HU&book=Temperature&all=1

How good is it?

- Possible to achieve an inaccuracy of <1% RH
- Sensitivity increases markedly as T increases, and slightly as the RH decreases
- Assmann psychrometer
  - http://www.climatronics.com/pdf_pn/Calibration_Test_Fixtures/225-5230.pdf
  - Can be used to check other instruments

Psychrometric chart

Tables are also available



http://en.wikipedia.org/wiki/Image:Psychrometric_chart_simplified.png

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

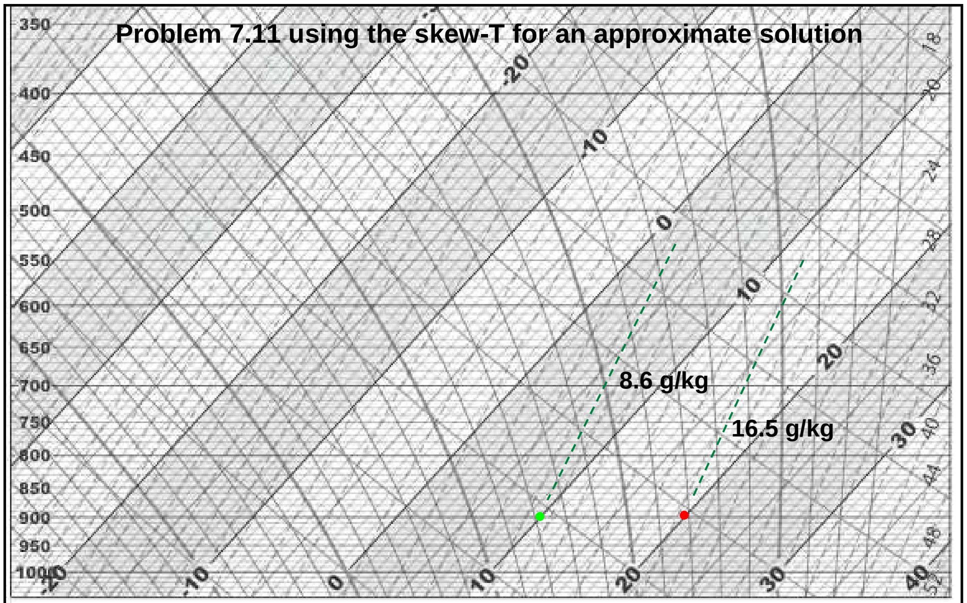
Measurement of relative humidity

Ex: Vaisala HMP-45C

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Problem 7.11 using the skew-T for an approximate solution



Given: $T = 20$ C
$T_d = 10$ C
p = 900 hPa

Find f
$f = r_v/r_{vs} = r_{vs}(T_d)/r_{vs}(T)$
$= 8.6 / 16.5$
$= 0.52$

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c) Equivalent temperature (T_{ie})

This is the temperature achieved via isobaric (p=const) condensation (latent heating) of all water vapor. T_{ie} is a fictitious temperature – there is no atmospheric process that is associated with it. (In fact Tsonis notes that T_{ie} is the reverse of an irreversible process associated with T_{iw}.) Thus, this is also referred to as the isobaric equivalent temperature (T_{ie}). This process is similar (but opposite) to that of the isobaric wet-bulb temperature, T_{iw}, so the same equation applies. In this case, integration of (6.7) gives

$$\int_{T}^{T_{ie}} dT = \frac{L_{vl}}{c_p} \int_{r_v}^{0} dr_v$$

or

$$T_{ie} = T + L_{lv} r_v / c_p. \qquad (6.9)$$

How does the isobaric equivalent temperature differ from the adiabatic equivalent temperature? What is the relation between adiabatic equivalent temperature and adiabatic equivalent potential temperature? [Brief discussion here.]

T_{ie} and T_{iw} are related by Eq. (6.7) and represent the respective maximum and minimum temperatures that an air parcel may attain via the isenthalpic (adiabatic and isobaric) process.

d) *Saturation point temperature (T_{sp})*

This is also called the "*isentropic condensation temperature*" (T_c) as defined by Bolton (1980), or the more classical temperature of the lifting condensation level (T_{lcl}). T_{sp} is achieved via adiabatic lifting (cooling by expansion). The value of T_{sp} is easily found graphically on a skew-T diagram (see Fig. 6.2).

Recall that the adiabatic equation can be derived from the First Law and equation of state to get

$$c_p dT = R_d T(dp/p).$$

Also recall that the integrated form is Poisson's equation

$$(T/T_0) = (p/p_0)^{\kappa} \qquad (6.10)$$

We now note that $T_{sp} = T_d(r_v, p_{sp})$. Substitution of (6.4), the expression for T_d, into (6.10) gives an iterative formula (derivation given in Rogers and Yau 1989):

$$T_{sp} = \frac{B}{\ln\left[\left(\frac{A\varepsilon}{r_v p_0}\right)\left(\frac{T_0}{T_{sp}}\right)^{1/\kappa}\right]}$$

More accurate (and explicit) empirical expressions are given by Eq. (21) in Bolton (1980):

$$T_{sp} = \frac{2840}{3.5\ln T - \ln e - 4.805} + 55. \qquad \text{(using vapor pressure e)}$$

What about a form that has mixing ratio as an input? Use $r_v = \varepsilon e/p \rightarrow e = r_v p/\varepsilon$

$$T_{sp} = \frac{1}{\frac{1}{T-55} - \frac{\ln(f/100)}{2840}} + 55 \qquad \text{(using relative humidity f)}$$

For these two formulations, T_{sp} is in °C, T in deg K, f in %, and e in mb. The graphical method of determining T_{sp} is known as *Normand's Rule* (p. 207 in Petty), illustrated in Fig. 6.2 (and Petty Fig. 7.16) (To be clarified in class.)

Back to Problem 7.9e:

We have been considering saturation by adiabatic expansion:

adiabatic expansion → cooling → tendency towards saturation

Taking the log differential of relative humidity, $f=e/e_s$ we get

$$d\ln f = d\ln e - d\ln e_s$$

Note that the ratio $e/p = N_v$ is constant during ascent, which is equivalent to saying that the mixing ratio $r_v = \varepsilon e/p$ is conserved. Furthermore, from Poisson's equation, $Tp^{-\kappa}$ is constant (i.e., θ is conserved). Since $e = N_v p$, then

$$Te^{-\kappa} = N_v^{-\kappa} \text{ x const} = \text{new const.}$$

Taking the log differential of the above, we obtain

$$d\ln T = \kappa d\ln e \text{ (or } d\ln e = \kappa^{-1} d\ln T\text{).}$$

Use the differential form of the C-C eq.

$$d\ln e_s = \frac{L_{vl}}{R_v T^2} dT$$

to write

$$d\ln f = \kappa^{-1} d\ln T - \frac{L_{vl}}{R_v T^2} dT$$

The first term on the RHS is the change due to a decrease in p (and e)

The second term represents the change in f from a decrease in T and $e_s(T)$.

These terms have opposite signs; therefore, adiabatic expansion could increase or decrease f.

To clarify this point, we can write the above to represent the slope, df/dT, the sign of which we want to determine:

$$\frac{df}{dT} = \frac{f}{T}\left(\kappa^{-1} - \frac{L_{vl}}{R_v T}\right) = \frac{f}{T}\left(\frac{c_p T - \varepsilon L_{vl}}{R_v T}\right)$$

In other words, we are asking, “How does f vary with T?”

This equation shows that df/dT < 0 (i.e., f increases when T decreases) when the condition

$c_pT < \varepsilon L_{vl}$ or $T < \varepsilon L_{vl}/c_p \cong 1500$ K.

Adiabatic expansion and condensation

Does adiabatic expansion necessarily produce condensation?
Consider the following example hypothetical problem:

Condensation of water can occur in updrafts because the saturation mixing ratio decreases in adiabatic ascent. This property of water can be attributed to the high value of latent heat of condensation. It has long been speculated that there may be trace gases which, because of low values of L, would condense in downdrafts (Bohrens 1986). Show that the criterion that must be satisfied if vapor is to condense in downdrafts (adiabatic compression) is
$L < c_pT/\varepsilon$.

Solution:
From the definition of $f=e/e_s$, we can write dlnf/dz = dlne/dz – dlne$_s$/dz. Since e = r_{vp}/ε (and mixing ratio r_v is constant), dlne = dlnp. Then dlnf/dz = dlnp/dz – (dlne$_s$/dT)(dT/dz). We now use the C-C equation dlne$_s$/dT = $L/(R_vT^2)$ and insert into the previous equation: df/dz = p^{-1}dp/dz – (L/R_vT^2)dT/dz. Recall that the dry adiabatic lapse rate (dT/dz here) is dT/dz = $-g/c_p$. Also, p^{-1}dp/dz = $p^{-1}g/\alpha = g/R_dT$. Substitution of these into the previous yield dlnf/dz = $g/R_dT - gL/(c_pR_vT^2)$ =
$g/(R_dT)[1 -LR_d/(c_pR_vT)] = g/(R_dT)[1-(L\varepsilon/c_pT)]$. Thus, if f increases with decreasing height, the term in brackets should be > zero, i.e., $1 - L\varepsilon/C_pT > 0$.
Rewriting, $L < c_pT/\varepsilon$ is the criterion for saturation upon descent. For the atmosphere, c_p = 1005, T = 290, and e = 0.622, we have $L < 4.7\text{x}10^5$ J kg^{-1}. This is clearly not satisfied for water, but is possible for some volatile substances.

Determination of cloud base from the dew point depression, (T-T_d)

There is a practical application that is closely assocated with T_{sp}.
In this application we will derive a relationship between the height of T_{sp} and the surface dewpoint depression, T-T_d.

Assume that a surface parcel rises (adiabatically) until condensation occurs (this defines a Cu cloud cloud base). In reality, r_v typically exhibits a negative vertical gradient, because the source of r_v is surface evaporation, and the sink is mixing from above.

The relation that we derive will provide a useful formula for estimation of the base of cumulus clouds, given a measurement of (T-T_d) at the surface.

Recall that the lapse rate for a subsaturated parcel is given by the dry adiabatic lapse rate (g/c_p), approximately 10 K km^{-1}. To estimate the height at which condensation occurs, we need to examine the variation of T_d along a dry adiabat. This is given by the C-C eq.

$$dT_d = \frac{R_v T_d^{\,2}}{L_{vl}} d\ln e$$

Using the relation dlnT = κdlne ($\kappa = R_d/c_{pd}$) we can rewrite the above as

$$dT_d \cong \frac{R_v T_d^{\ 2}}{\kappa L_{vl}} \frac{dT}{T} = \frac{c_p T_d^{\ 2}}{\varepsilon L_{vl}} \frac{dT}{T}$$

For T ≈ Td ≈ 273 K, and using finite differences, we obtain the approximate relation

$\Delta T_d \approx (1/6)\Delta T$,

i.e., the magnitude of the T_d decrease is about one sixth that of the adiabatic lapse rate for a parcel undergoing adiabatic ascent (see Fig. 6.3).

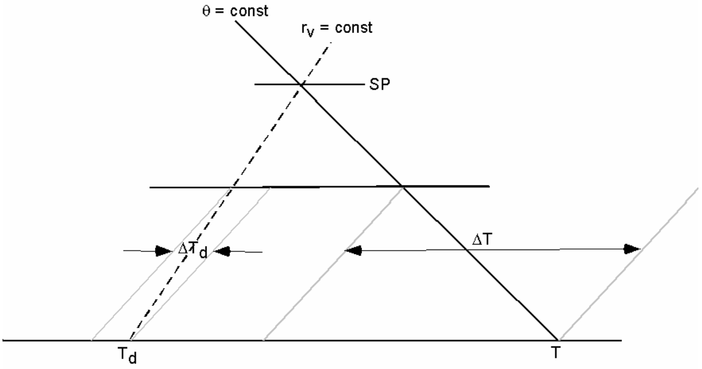


Fig. 6.3. *Illustration of the relation between decreases in T and T_d during adiabatic lifting of a subsaturated parcel*

This provides a basis for the following equations 7.29 and 7.30.
More on the saturation point (SP) or lifting condensation level (LCL)

Reference to Section 7.6 in Petty (p. 191)

We have developed a precise relationship for T_{sp}

Some approximate relationships for p_{sp} and z_{sp}:

$P_{sp} \approx p \bullet \exp(-0.044\Delta T_d)$ (7.29)

where ΔT_d is the dewpoint depression ($T - T_d$) in deg C.

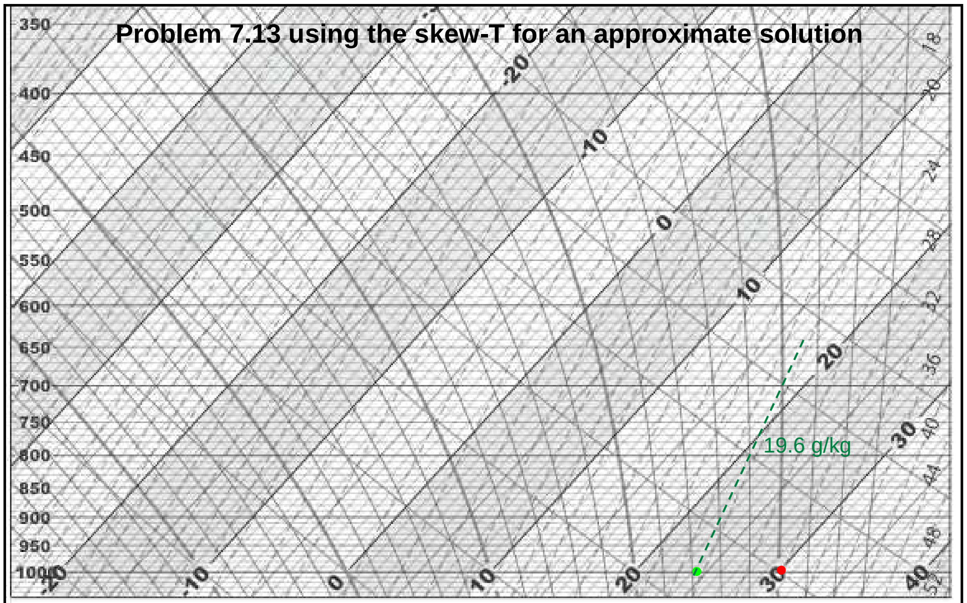
$z_{sp} \approx (T - T_d) / 8$ (7.30)

Problem 7.13
T = 30 C, f = 0.70
a) Find T_d: First, find r_v using $r_v = r_{vs}(0.70) = \varepsilon e_s(30\ C)(0.70)/p$
Then use Eq. (6.2) from my notes:
b) Find $z_{sp} = (T - T_d)/8$

$$T_d = T_d(r_{vs}, p) = \frac{B}{\ln\left(\frac{A\varepsilon}{r_v p}\right)}$$

This can be done with the skew-T (next slide)

Problem 7.13 using the skew-T for an approximate solution



T = 30 C
f = 0.70
r_{vs} = 28 g/kg (skew-T)
r_v = 28(0.70) = 19.6

Then T_d = 24.5 C (skew-T)

$Z_{sp} = (T-T_d) / 8 = 5.5 / 8 =$ 0.7 km

*Assumed p = 1000 hPa

---

*6.2.1 Derivation of the reversible saturated adiabatic lapse rate**

We have considered a related topic in the derivation of the pseudo adiabatic lapse rate and θ_e. In Chap. 3 (notes), we considered a preliminary form of the pseudo-adiabatic lapse rate, Eq. (3.21), reproduced here:

$$\left(\frac{dT}{dz}\right)_s \equiv \Gamma_s = \frac{\Gamma_d}{1+\dfrac{L_{vl}}{c_p}\dfrac{dr_{vs}}{dT}} \qquad (3.21)$$

The term in the denominator required the Clausius-Clapeyron Equation to provide an expression for dr_{vs}/dT.

This term is related to the magnitude of latent heating within the saturated parcel.

As shown in Fig. 6.4, the local lapse rate along the saturated adiabat in the lower right side (warm, high water vapor content) is relatively low, while at low pressure and cold temperature (upper part) the local value of the saturated adiabat approaches that of the dry adiabat.

* Corresponding material is in Petty, Section 7.7

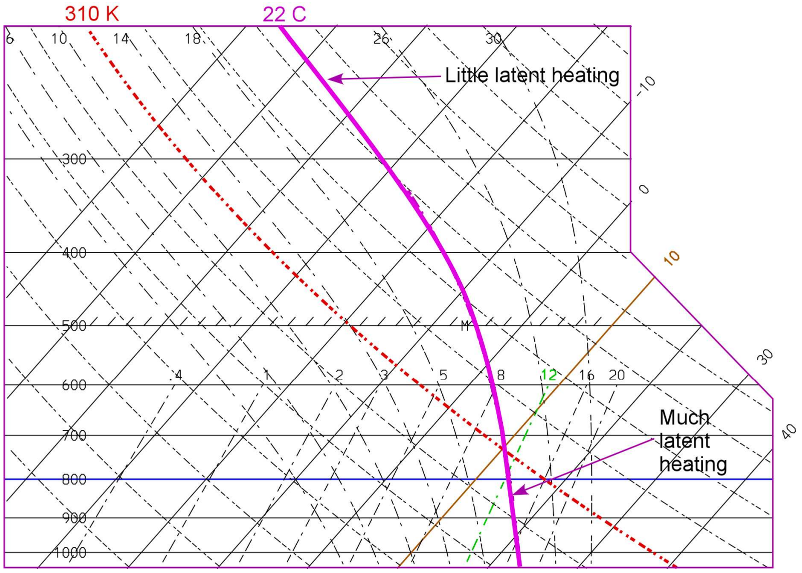


Figure 6.4. Variation in the local value of ∂T/∂z along the saturated adiabat (bold solid line) on the Skew-T, ln p diagram.

One can make two limiting assumptions regarding the condensed water:

i) It is carried along with the parcel.

ii) It immediately leaves the parcel (by removal – called sedimentation – from the precipitation process).

In reality, the clouds in the atmosphere are somewhere in between these two extremes.

The latter process is defined as the pseudoadiabtic process and simplifies matters since one need not consider the heat content of the condensed water (condensate) that is carried with the parcel. This process (and lapse rate) was considered in Section 3.11.3, Eq. 3.21 (above).

We also note that the pseudoadiabtic process is irreversible, whereas the saturated adiabatic process (i) is reversible. It turns out that the lapse rate defined by each is nearly the same, but the differences may be important in some situations.

The starting point differential equation (based on the First Law) for the reversible process has an extra term that expresses that amount of heat contained by the condensate:

$$c_w r_a dT - d(L_{vl} r_{vs}) = \left[c_{pd}(1 - r_{vs}) + c_{vv} r_{vs}\right] dT - dp/\rho \qquad (6.11)$$

where r_a is the weight of condensed water per gram of air (this will be referred as the adiabatic liquid water content).

Our starting point for the pseudo-adiabatic lapse rate was a simplified form of (6.11), already having the approximation L_{vl}=const. We also ignored the contribution of enthalpy from moist air, the second term within the brackets on the RHS of (6.11).

The key term here is the first term on the LHS, $c_w r_a dT$, the heat stored by the condensate.

Introducing the equation of state ($p=\rho R_d T$) and then expanding terms in (6.11) yields

$$c_w r_a dT - L_{vl} dr_{vs} - r_{vs} dL_{vl} = c_{pd} dT + r_{vs}(c_{vv} - c_{pd}) dT - R_d T \frac{dp}{p}$$

The final form (taken from Iribarne and Godson, 1973) is

$$\Gamma_{s-rev} = \Gamma_d \frac{\dfrac{p}{p - e_s}\left[1 + \left(\dfrac{L_{vl}}{R_m T} - 0.61\right) r_{vs}\right]}{1 + \dfrac{c_{pv} r_{vs} + c_w (r_{tw} - r_{vs})}{c_{pd}} + \dfrac{L_{vl}^2 r_{vs} (\varepsilon + r_{vs})}{c_{pd} R_d T^2}} \qquad \text{A1}$$

where $R_m = R_d(1+0.61r_v)$, the total water mixing ratio $r_{tw} = r_{vs} + r_a$ (r_a is the adiabatic mixing ratio of condensed water), and $\Gamma_d = g/c_{pd}$.

Refer to the handout, copied from Chapter 6 of the book *Atmospheric Thermodynamics* (Bohren and Albrecht (1998) for details of the derivation.

The formula for the pseudoadiabatic lapse rate is (assuming that no liquid water remains with the parcel, i.e., r_{tw} - r_{vs} = r_a = 0 in the above equation).

$$\Gamma_{s-pseudo} = \Gamma_d \frac{\frac{p}{p-e_s}\left[1+\left(\frac{L_{vl}}{R_m T}-0.61\right)r_{vs}\right]}{1+\frac{c_{pv} r_{vs}}{c_{pd}}+\frac{L_{vl}{}^2 r_{vs}(\varepsilon + r_{vs})}{c_{pd} R_d T^2}} \qquad \text{A2}$$

One can also make the following approximations in Eq. A2:

$$\frac{p}{p-e_s} \approx 1 \qquad \left(\frac{L_{vl}}{R_m T}-0.61\right) \approx \frac{L_{vl}}{R_m T} \approx 8.7 \qquad 1+\frac{c_{pv} r_{vs}}{c_{pd}} \approx 1$$

to get the approximate form for the pseudoadiabatic lapse rate (also letting $R_d \approx R_m$):

$$\Gamma_{s-approx} = \Gamma_d \frac{1+\frac{L_{vl} r_{vs}}{R_d T}}{1+\frac{\varepsilon L_{vl}{}^2 r_{vs}}{c_{pd} R_d T^2}} \qquad \text{A3}$$

Compare with Eq. 6.111 from Bohren & Albrecht

This is an approximation for the local slope and should never be used be used for calculations requiring an accurate parcel T, since the errors will accumulate to unacceptably high values in the integration.

Example (from Iribarne and Godson, p. 158):

This will illustrate the errors for a saturated parcel with the following values:
T = 17 °C, p = 1000 mb, e_s = 19.4 mb, r_{vs} = 0.0123 kg kg^{-1}.
Also assume that r_a = 0.004 kg kg^{-1}.

Equation A1, A2 and A3 yield the following values:

$\Gamma_{s\text{-}rev} = 4.40$ K km^{-1}

$\Gamma_{s\text{-}pseudo} = 4.42$ K km^{-1}

$\Gamma_{s\text{-}approx} = 4.50$ K km^{-1}

These differences are appreciable.

Another example take from Emanuel (1994, p. 133) illustrates how the differences can accumulate, as shown below in Table A1. Also refer to Bohren and Albrecht, Fig. 6.3, reproduced below.

Table A1. *Parcel temperature obtained from $\Gamma_{s\text{-}rev}$ (Eq. A1) and $\Gamma_{s\text{-}pseudo}$ (Eq. A2), for a saturated parcel ascending with initial conditions p = 950 mb, T = 25 C (very wet) (From Emanual 1994)*

p (mb)	$T_{s\text{-}rev}$ (K)	$T_{s\text{-}pseudo}$ (K)	$T_{s\text{-}rev}$-$T_{s\text{-}pseudo}$ (K)	$T_{\rho\text{-}rev} - T_{\rho\text{-}pseuo}$
950	298.15	298.15	0	0
800	292.36	292.35	0.01	-1.04
700	287.77	287.73	0.04	-1.75
600	282.32	282.22	0.10	-2.44
500	275.59	275.36	0.23	-3.07
400	266.78	266.27	0.51	-3.51
300	254.10	252.90	1.21	-3.39
200	233.30	230.32	2.98	-1.76
100	195.77	189.96	5.81	1.70

The right-most column in Table A1 is the difference between density temperatures (T_ρ) for the two processes. Density temperature is the temperature dry air would have to yield the same density as moist, cloudy air (i.e., virtual temperature including water vapor and cloud condensate – more on this later).

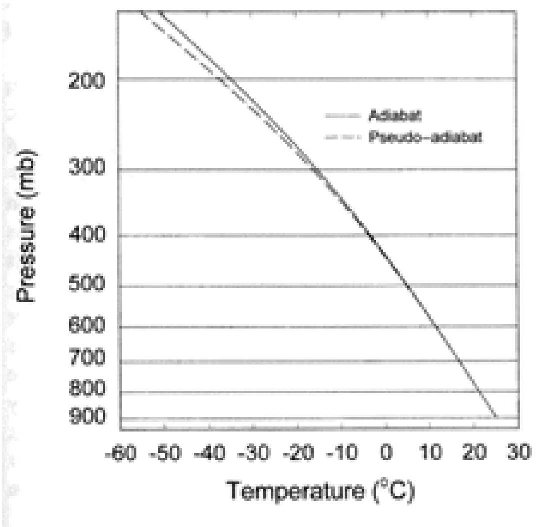


Figure 6.3 Temperature as a function of pressure for adiabatic and pseudoadiabatic ascent from initial conditions of 900 mb at 25°C. For adiabatic ascent the total water mixing ratio w_t is fixed at the saturation value for 900 mb and 25°C.

Notes:

Perhaps the most common application involves the calculation of convective available potential energy (CAPE), which requires an accurate calculation of the parcel T_s during saturated ascent. CAPE is discussed in Chap. 8 of the Notes and pp. 263-265 of Petty.

If we recognize that θ_e is conserved along a saturated adiabat, then a numerical calculation of T_s should conserve θ_e. This can be used as a check for the accuracy of the T_s calculation.

After having considered the differences between the reversible and pseudo saturated adiabatic processes, we can ask the question “Which is the best to use?” We cannot fully answer this question until we have a greater understanding of cloud physics and microphysical structure of clouds.

Effects of freezing. If we use the latent heat of deposition, then a difference will exist between Γ_s in Eq. A3 above for condensation and deposition. This difference is shown in the figure below.

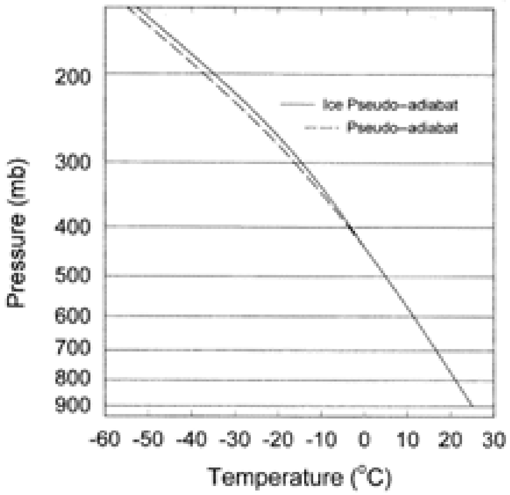
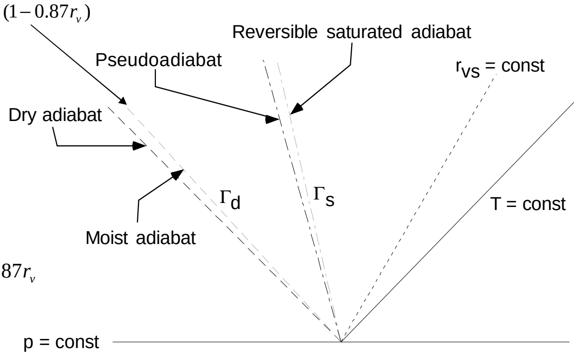


Figure 6.4 Temperature as a function of pressure for pseudoadiabatic ascents calculated assuming no freezing of liquid water (dashed line) and freezing of all liquid water at temperatures below 0°C (solid line). The initial state of the parcel is the same as that for Fig. 6.3.

Relative orientation or slope (magnitude) of dry and saturated adiabats on the skew-T diagram. (Fig. 6.7)
The magnitude of the saturated adiabatic lapse rate is slightly less than that of the pseudoadiabatic lapse rate.
The magnitude of the dry adiabatic lapse rate (for dry air) is slightly greater than its counterpart for moist (rv nonzero) air, since the effective c_p is larger [$c_p(1+0.887r_v)$].

$$\Gamma_m = \frac{g}{c_{pm}} = \frac{g}{c_{pd}(1+0.87r_v)} = \frac{\Gamma_d}{1+0.87r_v} \cong \Gamma_d(1-0.87r_v)$$



The ratio of the dry to moist adiabatic lapse rates is

$$\frac{\Gamma_m}{\Gamma_d} = \frac{g/c_{pd}}{g/c_{pd}(1+0.87r_v)} = \frac{1}{1+0.87r_v} \cong 1-0.87r_v$$

For a mixing ratio of 20 g kg-1, this ratio is 0.983.

Thus, a difference up to ~2%, or 0.2 K km-1, is possible. This is potentially significant!

6.2.2 Adiabatic liquid water content (see also p. 222 in Petty)

A measure of water condensed along the saturated adiabat.
Expressed in g m^{-3} by multiplying r_{ca} by the density, ρ, of moist air.

Typical values of r_c in clouds are in the range 0.5 to 3 g m^{-3}.

During saturated adiabatic ascent, a reduction in water vapor ($-dr_{vs}$) is a gain in cloud condensate (r_c), such that the total water is conserved. Thus,

$$d\chi = \rho dr_{ca} = -\rho dr_{vs}. \qquad (6.12)$$

χ can be derived from the adiabatic equation. It is related to the difference between the dry and saturated adiabatic lapse rates (this is a homework problem).

χ is easily found graphically as shown in the figure below.

Its value requires knowledge of cloud base (saturation point) conditions (r_{vs},p) and is computed as

$$\chi = \rho_m [r_{vs}(T_{sat},p)_{\theta e=const} - r_{vs}(T_{sp},p_{sp})]$$

T_{sat} is evaluated along a saturated adiabat (line of constant θ_e).

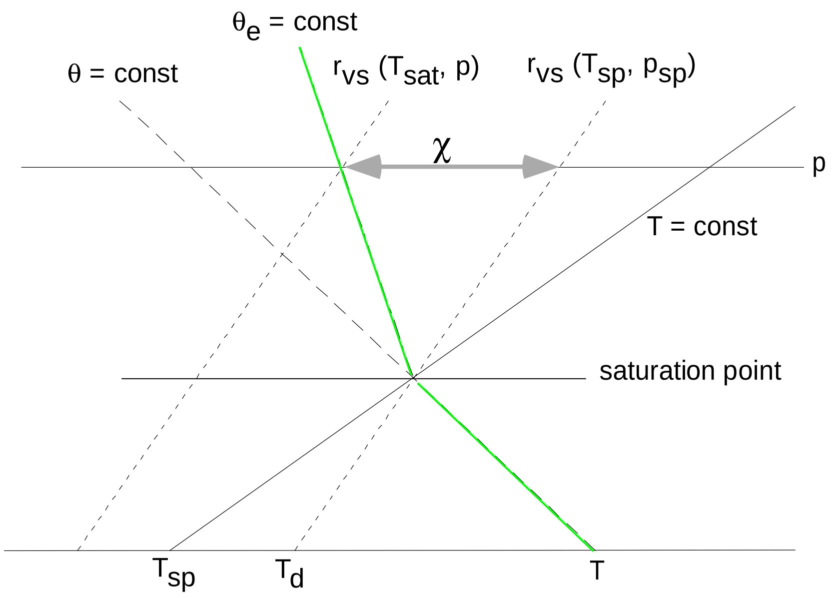


Figure 6.4. Graphical determination of adiabatic liquid water content.

Adiabatic liquid water content illustration from Petty

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

6.2.3 Relation between θ_e and θ_w

θ_e was derived in increasing levels of accuracy in this chapter, and in Chap. 3.

One can determine θ_w if T_w is known, and then calculate the temperature of the parcel after it descends along a *saturated* adiabat to 1000 hPa (mb).

In a sense, this process is the opposite of that by which θ_e is achieved, where the parcel ascends along a saturated adiabat until all water vapor is removed by condensation.

This numerical computation of either of these processes would require use of Eq. A1 or Eq. A2.

The derivation of θ_w begins with the assumption that the parcel is at its saturation point.

We start with the differential equation

$$c_p d \ln \theta = d\left(\frac{L_{vl} r_v}{T}\right)$$

and integrate

$$c_p \int_{\theta}^{\theta_w} d \ln \theta = \int_{r_v}^{r_{vs}(\theta_w)} d\left(\frac{L_{vl} r_v}{T}\right)$$

The result is

$$\theta_w = \theta \exp\left(\frac{L_{vl}}{c_p}\left(\frac{r_v}{T_{sp}} - \frac{r_{vs}(\theta_w)}{\theta_w}\right)\right) \qquad \text{(A5)}$$

This is another transcendental equation that requires iteration. Since the exponent term is less than ~0.2, (A5) can be approximated as

$$\theta_w \approx \theta + \left(\frac{L_{vl}}{c_p}\left(\frac{\theta}{T_{sp}} r_v - \frac{\theta}{\theta_w} r_{vs}(\theta_w\right)\right) \qquad \text{(A6)}$$

If we can further assume that the ratios θ/T_{sp} and θ/θ_w are near unity, then further approximation provides the following:

$$\theta_w \approx \theta + \left(\frac{L_{vl}}{c_p}(r_v - r_{vs}(\theta_w)\right) = \theta_e - \frac{L_{vl}}{c_p} r_{vs}(\theta_w) \qquad \text{(A7)}$$

This reveals the relationship between θ_w and θ_e, both of which are conserved for saturated adiabatic processes

Heat Index (HI) and thermodynamics

Good descriptions:

a) http://en.wikipedia.org/wiki/Heat_index

b) http://www.crh.noaa.gov/jkl/?n=heat_index_calculator

c) http://www.weatherimages.org/data/heatindex.html

Is there any correlation between HI and T_{ie}*?*

Extra credit (25 pts): Create a scatter plot of HI vs. T_{ie} *(or some other appropriate thermodynamic variable) for a range of temperatures and RH (e.g., T = 80, 85, 90, 95 and RH = 30, 40, 50, 60, 70, 80, 90% for each T)*

6.2.4 The wet equivalent potential temperture (θ_q)

See RY, pp. 25-26 for derivation.

θ_q is conserved for saturated processes, and has been used to analyze mixing processes in clouds.

More on applications of this will be considered later.

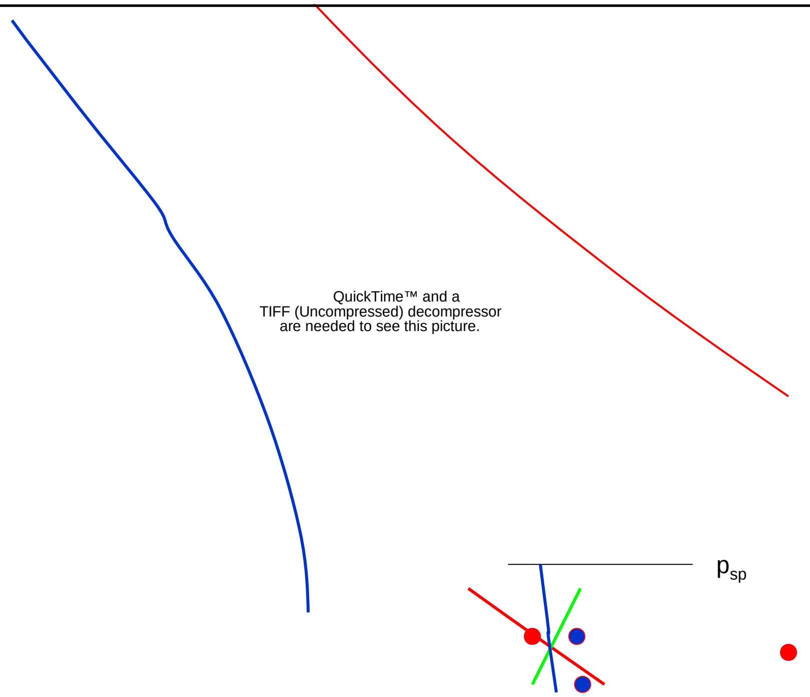
Mixed layers (as in a convective boundary layer)

a) Unsaturated mixed layer (Fig. 7.24b,c)
b) Cloud-topped mixed layer (Fig. 7.24d)

6.4 Example computations using the skew-T diagram

Given: p=900 hPa, T = 25 °C, T_d = 15 °C (variables commonly reported)

Find: r_v,
f,
θ,
T_v,
T_{sp},
T_e,
T_{ae},
θ_e,
T_w,
θ_w,
χ(at 50 kPa)



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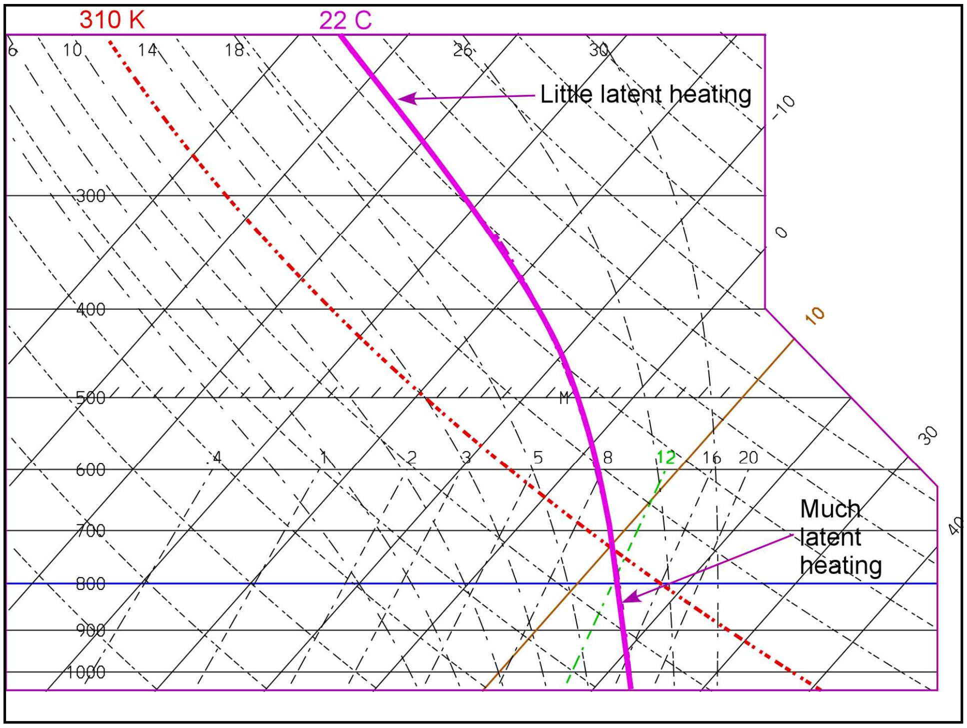

Little latent heating

Much latent heating

The following figure and table (i) summarize the relative magnitudes of the basic and derived thermodynamic variables that we have considered to this point and (ii) illustrate their graphical determination. The non-potential temperatures exhibit the inequality:

$$T_{sp} < T_d < T_w < T < T_v < T_{ie} < T_{ae}$$

Variable	Process			
	Isbaric warming or cooling (without condensation or evaporation) such as radiation	Isobaric evaporation or condensation	Non-saturated adiabatic expansion or compression	Saturated adiabatic expansion or compression (reversible)
f	NC	NC	NC	C
e or T_d	C	NC	NC	NC
r_v	C	NC	C	NC
T_w or T_{ae}	NC	C	NC	NC
θ	NC	NC	C	NC
θ_e or θ_w	NC	C	C	C
T_{sp}	NC	NC	C	C
T	NC	NC	NC	NC

NC – not conserved; C – conserved

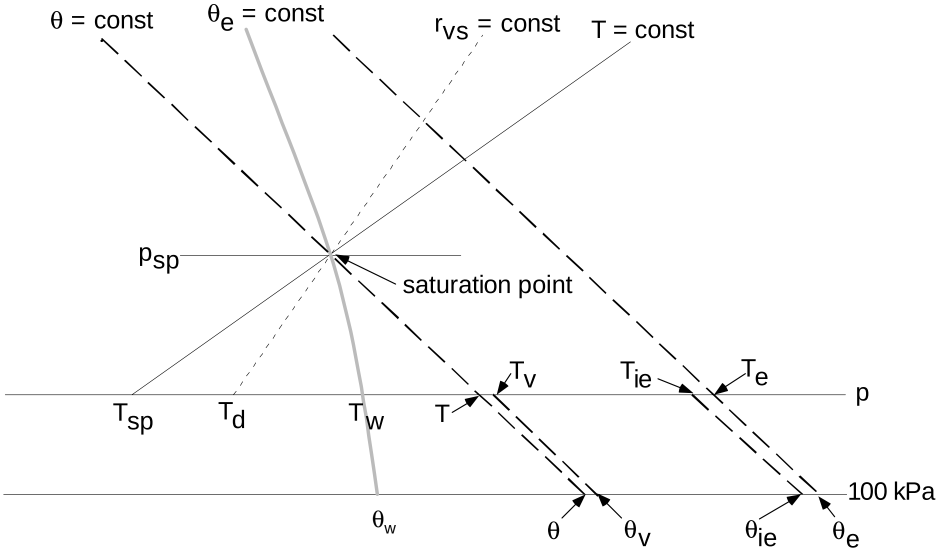

Figure 6.5. Summary of temperature/humidity parameters, plotted on a tephigram. Line orientations are similar for a skew-T, ln p diagram. (After Iribarne and Godson, 1973)

Example problems:

From Petty:

7.13

7.14

7.16

7.17

7.22

Problem 7.14: The definition (7.55) of equivalent potential temperature assumes that all condensation results in the appearance of liquid water, regardless of the temperature. If freezing occurs, additional latent heat is released, which further raises the temperature of the parcel. Assume that a parcel is at saturation at a temperature of freezing and a pressure of 900 hPa.
a) What is the equivalent potential temperature θ_e according to the traditional definition?
b) By how much would θ_e increase if all condensate were assumed to freeze? Answer: 1.5 K

See Eq. 7.50 (p. 203) and make the correction
Assume that the condensate freezes instantaneously at a fixed pressure
Then

$$dq = -L_{li} dr_{vs} = c_p dT$$

Note: The value of r_{vs} at 0 C and 900 mb is about 4.3 g kg^{-1} = 0.0043 kg kg^{-1}

Integrate and solve for ΔT:

$$\Delta T = -L\Delta r_{vs} / c_p = -3.34 \times 10^5 \text{ J kg}^{-1} (-0.0043) / 1009.5 \text{ J K}^{-1} \text{ kg}^{-1}$$
$$= 1.42 \text{ K}$$

But note: $c_{pm} = c_{pd} (1 + 0.887 r_v) = 1005.7 (1+0.887*0.00043) = 1009.5$

Problem 7.17: The equivalent potential temperature of a parcel is 310 K. (a) Graphically, estimate its wet-bulb potential temperature θ_w. (b) If its potential temperature q is 300 K, graphically find its mixing ratio w.

Do we need to assume a pressure??? No. The pressure is implicitly given.

(a) Use a skew-T and find the saturated adiabat corresponding to θ_e = 310 K. Then find the value of T along this adiabat at p = 1000 mb.

(b) Find the dry adiabat corresponding to θ = 300 K. The intersection of this dry adiabat with the saturated adiabat found in part (a) determines the SP of this parcel, and hence the value of r_v (w).

$$\theta_w \approx \theta + \left(\frac{L_{vl}}{c_p} \left(r_v - r_{vs}(\theta_w) \right) \right) = \theta_e - \frac{L_{vl}}{c_p} r_{vs}(\theta_w)$$

Homework:

- Problems 1-5 (my notes)