5.2 Snow crystal and snowflake size distributions

- Similar to MP48, Gunn and Marshall (1958) proposed an exponential size distribution for aggregates of snow crystals based on observations (right)

\[ N(D) = N_0 \exp(-\Lambda D), \quad [1a] \]

where \( \Lambda = 25.5 R^{-0.48} \text{ cm}^{-1}, \quad N_0 = 3.8 \times 10^3 R^{-0.87} m^{-3} \text{ mm}^{-1}, \quad [1b] \)

and D is the equivalent diameter of the water drop to which the ice crystal aggregate melts (i.e., equivalent melted drop diameter) and R is the rate of precipitation in mm h\(^{-1}\) of liquid water

- Variety of snow and snowflake types, including aggregates of plate, columns and dendrites

- Note typo error in units of \( \Lambda \) in Pruppacher and Klett (1997)

- Sekhon and Srivastava (1970) found similar form as [1a] in snow except different equations for \( \Lambda, N_0 \) in [1b]

  - \( \Lambda = 22.9 R^{-0.45} \text{ cm}^{-1} \)
  - \( N_0 = 2.50 \times 10^3 R^{-0.94} \text{ mm}^{-1} \text{ m}^{-3} \)

- SS70 also derive several useful relationships for snow

  - Median volume diameter: \( D_0 = 0.14 R^{0.45} \text{ (cm)} \)
  - Liquid water content \( W = 0.250 R^{0.86} \text{ (g m}^{-3}\)
  - Reflectivity: \( Z = 1780 R^{2.21} \text{ (mm}^6 \text{ m}^{-3}\)

---

Gunn and Marshall (1958)
5.3 Snow crystal and snowflake density

- Most ice crystals, and all aggregates of ice crystals, have a bulk density less than that of solid ice (0.916 g cm\(^{-3}\))
  - Small amounts of air in capillary spaces of single crystals (e.g. hollow columns)
  - Tendency of single snow crystals to grow in skeletal fashion (e.g., dendrites)
  - Obvious air gaps when multiple snow crystals aggregate

For most snow crystal types, increasing size implies decreasing bulk ice density

**TABLE 2.3**

Bulk density of various snow crystals (\(d\) and \(L\) in mm). (Based on data of Heymsfield, 1972.)

<table>
<thead>
<tr>
<th>Crystal type</th>
<th>Bulk Density, (\rho_c), (g cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>hexagonal plate</td>
<td>(\rho_c = 0.9)</td>
</tr>
<tr>
<td>plates with dendritic extensions</td>
<td>(\rho_c = 0.656 \ d^{-0.627})</td>
</tr>
<tr>
<td>dendrites</td>
<td>(\rho_c = 0.588 \ d^{-0.377})</td>
</tr>
<tr>
<td>stellar, broad arms</td>
<td>(\rho_c = 0.588 \ d^{-0.377})</td>
</tr>
<tr>
<td>stellar, narrow arms</td>
<td>(\rho_c = 0.46 \ d^{-0.482})</td>
</tr>
<tr>
<td>column, cold region</td>
<td>(\rho_c = 0.65 \ L^{-0.0915})</td>
</tr>
<tr>
<td>column, warm region</td>
<td>(\rho_c = 0.848 \ L^{-0.014})</td>
</tr>
<tr>
<td>bullet</td>
<td>(\rho_c = 0.78 \ L^{-0.0038})</td>
</tr>
</tbody>
</table>

Pruppacher and Klett (1997)
From Heymsfield (1972)
Integrated bulk density and size/aspect ratio information for various *snow crystal types*

For plates and dendrites

\[
\rho = a_1 D^c \quad [3a]
\]

\[
h = a_2 D^f \quad [3b]
\]

\(\rho\): bulk ice density (g cm\(^{-3}\))

\(D\): crystal diameter (cm) (major dimension)

\(h\): crystal thickness (cm) (minor dimension)

For columns, needles, and bullets

\[
\rho = a_1 L^c \quad [4a]
\]

\[
d = a_2 L^f \quad [4b]
\]

\(\rho\): bulk ice density (g cm\(^{-3}\))

\(D\): crystal length (cm) (major dimension)

\(d\): crystal thickness (cm) (minor dimension)

*Coefficients in table below (Matrosov et al. 1996)*

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**February 1996**

Matrosov et al.

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<table>
<thead>
<tr>
<th>Crystal class</th>
<th>(a_2)</th>
<th>(f)</th>
<th>(a_1)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dendrites, P1e</td>
<td>0.009</td>
<td>0.377</td>
<td>0.25</td>
<td>-0.377</td>
</tr>
<tr>
<td>Solid thick plates, C1g</td>
<td>0.138</td>
<td>0.778</td>
<td>0.916</td>
<td>0.0</td>
</tr>
<tr>
<td>Hexagonal plates, P1a</td>
<td>0.014</td>
<td>0.474</td>
<td>0.916</td>
<td>0.0</td>
</tr>
<tr>
<td>Solid columns, Cle ((L/d \leq 2))</td>
<td>0.578</td>
<td>0.958</td>
<td>0.916</td>
<td>0.0</td>
</tr>
<tr>
<td>Solid columns, Cle ((L/d &gt; 2))</td>
<td>0.260</td>
<td>0.927</td>
<td>0.916</td>
<td>0.0</td>
</tr>
<tr>
<td>Hollow columns, Cif ((L/d \leq 2))</td>
<td>0.422</td>
<td>0.892</td>
<td>0.53</td>
<td>-0.092 cold region</td>
</tr>
<tr>
<td>Hollow columns, Cif ((L/d &gt; 2))</td>
<td>0.263</td>
<td>0.930</td>
<td>0.53</td>
<td>-0.014 warm region</td>
</tr>
<tr>
<td>Long solid columns, N1e</td>
<td>0.035</td>
<td>0.437</td>
<td>0.916</td>
<td>0.0</td>
</tr>
<tr>
<td>Solid bullets, Cle ((L \leq 0.03 \text{ cm}))</td>
<td>0.153</td>
<td>0.786</td>
<td>0.916</td>
<td>0.0</td>
</tr>
<tr>
<td>Hollow bullets, Cld ((L &gt; 0.03 \text{ cm}))</td>
<td>0.063</td>
<td>0.532</td>
<td>0.77</td>
<td>-0.0038</td>
</tr>
<tr>
<td>Elementary needles, Nla ((L &lt; 0.05 \text{ cm}))</td>
<td>0.030</td>
<td>0.611</td>
<td>0.916</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Matrosov et al. (1996) from various sources in caption.
• **Bulk density of snow aggregate** \((\rho, \text{ g cm}^{-3})\) as function of aggregate diameter \((D, \text{ cm})\) (or major dimension) was provided by Passarelli and Srivastava (1979) based on Magono and Nakamura (1965) data

\[
\rho = 0.015D^{-0.6} \text{ [g cm}^{-3}\text{]} \quad [5]
\]

• Illingworth (1994), Matrosov et al. (1996) and Ryzhkov et al. (1998) recommend relationship for **bulk density of snow aggregates** \((\rho, \text{ g cm}^{-3})\) in terms of the ice particle major dimension \(S\) (mm) (note units!)

\[
\begin{align*}
\text{if } (S < 0.097 \text{ mm}) \text{ then } & \quad [6] \\
\rho &= 0.916 \text{ g cm}^{-3} \\
\text{else} & \\
\rho &= 0.07S^{-1.1} \text{ g cm}^{-3} \\
\text{endif}
\end{align*}
\]

• Equations [5] and [6] are in OK agreement for major dimension > 1 mm. Likely large variability in bulk ice density of aggregates under differing conditions
5.4 Snow crystal and snowflake orientation and refractive index

- In radar studies, usually assume major dimension of ice crystal or aggregate is in the **horizontal unless there is a strong electric field**, in which case ice particle is aligned with electric field, which is usually assumed strongest in **vertical** (Weinheimer and Few 1987)
- For dry ice particles (i.e., not in wet growth or melting), the refractive index is calculated with Debye theory using the bulk density of ice from earlier section. Recall...
- Use Debye mixing theory, Debye (1929), for ice and air mixtures (e.g., Battan 1973)

\[
\frac{K}{\rho} M = \frac{K_i}{\rho_i} M_i + \frac{K_a}{\rho_a} M_a \quad [7]
\]

\[
K = \frac{m^2 - 1}{m^2 + 2} \quad [8]
\]

- Where M: mass, \(\rho\): density, m: refractive index; subscript i=ice (solid) and a=air (no subscript=mixture or bulk ice density)
- Can simplify [7] by noting that \(m_a\) in [8] is \(\approx 1\) so \(K_a \approx 0\) and \(M \approx M_i \therefore \rightarrow K/\rho\) is constant. Hence, \(K\) for mixture is

\[
K = \left(\frac{K_i}{\rho_i}\right) \rho \quad [9]
\]

- Combine [4] and [5] to solve for refractive index of mixture (m)

\[
m^2 = \frac{2 \chi + 1}{1 - \chi} \quad \text{where} \quad \chi = \left(\frac{K_i}{\rho_i}\right) \rho \quad [10]
\]
5.5 Some basic polarimetric radar signatures of ice crystals and aggregates

- Employ the size, shape, density and orientation assumptions to look at some typical polarimetric radar quantities of ice crystals and aggregates

- Ice Crystal Type: **Hexagonal Plates (P1a)**
- PSD: Exponential
  - \( N(D) = N_0 \exp(-3.67D/D_0) \)
  - \( 2 \times 10^5 \text{ m}^{-3} \text{ cm}^{-1} \leq N_0 \leq 2 \times 10^6 \text{ m}^{-3} \text{ cm}^{-1} \)
  - \( 0.03 \leq D_0 \leq 0.07 \text{ cm} \ (D_{\text{max}} = 0.11 \text{ cm}) \)
- Shape: Model as Oblate Spheroid
  - \( a/b \) (minor:major): Auer and Veal (1970)
- Ice Density: Heymsfield (1972)
- Ice Orientation: Gaussian with mean in **horizontal** (0°) and varying standard deviation (0°, 30°)
- Radar Scattering and Propagation Model
  - T-matrix for oblate (e.g., Bringi and Chandrasekar 2001)
  - Mueller matrix for hydrometeor mixtures, PSD, canting, radar elevation angle etc (Vivekanandan et al. 1991)
  - C-band (5.5625 GHz, 5.33 cm)
  - 0° elevation angle
• Ice Crystal Type: **Columns (C1f, hollow, elongated, warm)**

• PSD: Exponential
  – \( N(D) = N_0 \exp(-3.67D/D_0) \)
  – \( 2 \times 10^5 \text{m}^{-3} \text{cm}^{-1} \leq N_0 \leq 2 \times 10^6 \text{m}^{-3} \text{cm}^{-1} \)
  – \( 0.03 \leq D_0 \leq 0.07 \text{cm} \) (\( D_{\text{max}} = 0.11 \text{cm} \))

• Shape: Model as Oblate Spheroid
  – \( a/b \) (minor:major): Auer and Veal (1970)

• Ice Density: Heymsfield (1972)

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  – **C-band (5.5625 GHz, 5.33 cm)**
  – 0° elevation angle

• **Z_{dr}** and **K_{dp}** for columns less than for plates. Why?
• How are **Z_{dr}** and **K_{dp}** related to \( N_0 \) and \( D_0 \)? More later…
- **Ice aggregates** (oblate), exponential PSD
  - \( N(D) = N_0 \times \exp(-3.67 \frac{D}{D_0}) \)
  - \( 1 \times 10^4 \text{ m}^{-3} \text{ cm}^{-1} \leq N_0 \leq 1 \times 10^5 \text{ m}^{-3} \text{ cm}^{-1} \)
  - \( 0.3 \leq D_0 \leq 0.7 \text{ cm} \quad (D_{\text{max}} = 1.5 \text{ cm}) \)
- **Ice density**: Illingworth (1994)
- **Shape**: \( a/b = 0.3 \) or 0.8
- **Orientation**: Gaussian distribution of ice particle canting angles
  - Mean: Horizontal (0°)
  - \( \sigma \) (std dev) canting angle was set at 5° (slight) or 30° (moderate).
- **Radar Scattering and Propagation Model**
  - T-matrix for oblate (e.g., Bringi and Chandrasekar 2001)
  - Mueller matrix for hydrometeor mixtures, PSD, canting, radar elevation angle etc (Vivekanandan et al. 1991)
  - C-band (5.5625 GHz, 5.33 cm)
  - 0° elevation angle

- More oblate \( (a/b=0.3) \) and less canted \( (\sigma=5°) \) particles have larger signatures.
- \( K_{dp} \) and \( Z_{dr} \) responses to H- or V-oriented low density aggregates are significantly less than most pristine ice crystals (primarily because of low density). Shape and orientation secondary.
K\textsubscript{dp} and Z\textsubscript{dr} Dependence on Mean Ice Particle Canting Angle

- Example: Plate (oblate spheroid), C-band
  - N\textsubscript{0} = 2\times10^6\text{m}^{-3}\text{ cm}^{-1}
  - D\textsubscript{0} = 0.07 cm
- Lack of mirror symmetry of radar parameters about 45° mean canting angle (e.g., Zdr/Kdp at 0° vs. 90°) related to random orientation of oblate in 2\textsuperscript{nd} angular (\(\phi\)) direction.
  - Reasonable at low elevation angle since ice would have no preferred orientation in this direction unless there is a strong horizontal field providing a preferred orientation.
- Electric Field (E-field) strength, particle size, particle shape, and particle density (among other ice properties) will determine mean canting angle.
  - Weinheimer and Few (1987)
  - More later...

K\textsubscript{dp} and Z\textsubscript{dr} are function of mean canting angle, which will depend on Electric field.
K\textsubscript{dp} and Z\textsubscript{dr} Dependence on Radar Elevation Angle

- Ice Crystal Example: Plate (oblate), C-band
  - N\textsubscript{0} = 2\times10\textsuperscript{6} m\textsuperscript{-3} cm\textsuperscript{-1}
  - D\textsubscript{0} = 0.07 cm
  - Vertically Oriented (90°)

- Ice orientation signatures in dual-polarimetric variables are insensitive to radar elevation angle up to about 15°
  - Decrease in magnitude of ice orientation signatures above 15° is apparent and could have an impact on even qualitative inferences, especially with weak signatures.
  - Consistent with well known results in radar community (e.g., associated with rain studies).

Dual-pol, including ice orientation signatures, insensitive to changes in radar elevation angle up to 15°.