Vertical Variability of the Raindrop Size Distribution and Its Effects on Dual-polarimetric Radar QPE

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Weather radar is a more economical means than rain gauges for rainfall mapping across large watersheds. However, even operational dual-polarimetric radars suffer from reduced accuracy with distance from the radar, largely due to the vertical variability of precipitation. To mitigate this drawback, we exploited characteristics of the measured precipitation profile to improve radar QPE at distant range.
Questions to Answer

1. How do ML characteristics impact the physics of rainfall in the vertical column?
2. Can the ML be used to improve radar-based QPE?
3. What are VP3 characteristics and how do they vary with rainfall intensity?
4. What are the impacts of VP3 variability on radar QPE?
Radar reflectivity factor: Size, D, and number concentration, N(D)

\[ Z_{h,v} = \frac{\lambda^4}{\pi^5 |K|^2} \int_0^{\infty} \sigma_{h,v}(D)N(D)dD \approx \int_0^{\infty} N(D)D^6 dD \]

Differential reflectivity: Reflectivity-weighted mean axis ratio, r

\[ Z_{DR} = 10 \log_{10} \left( \frac{Z_h}{Z_v} \right) \approx \langle r^{-7/3}_Z \rangle \]

Co-polar Correlation Coefficient: Variability of hydrometeor type, size, shape

\[ \rho_{hv} = |\rho_{co}| = \frac{|R_{co^*}|}{\sqrt{P_{co}^h P_{co}^v}} \]

Specific differential phase: Liquid water content, W, and mass-weighted axis ratio

\[ K_{DP} = \frac{180}{\pi \lambda} \int \text{Re}[f_h(D) - f_v(D)]N(D)dD \approx \frac{0.18}{\lambda} CW(1 - \bar{r}_m) \]
Rainfall estimators:
- R(Z) or Z-R
- R(K_{DP})
- R(Z_h, Z_{DR})
- R(Z_h, Z_{DR}, K_{DP})

- R(Z) and R(Z_h, Z_{DR}) most often types used for stratiform (e.g., Cifelli et al. 2011)

- Range-dependent biases arise due to variability in the measured precipitation profile

from Giangrande and Ryzhkov (2008)
Background: Dual-pol Radar Measured Precipitation Profiles

Dual-pol radar profiles

Melting Layer

Particle Images

from Hagen et al. (1993)
Question 1

How do ML characteristics impact the physics of rainfall in the vertical column?

Hypothesis:
A thicker and lower ML results in larger raindrops.
RHI scans from ARMOR and NPOL (RFloodS) were used to extract vertical profiles over 2DVD sites.

- Scans repeated every 1-2 min.
- Vertical resolution was 50 to 100 m.
- Melting layer (ML) mapped using RUC 0°C, $\rho_{hv}$ and $Z_e$ bright band.
Extraction of Vertical Profiles: Precipitation Trails

Range-height Scan from ARMOR

Extracted Profile

→ Following Precip Trail leads to thicker ML
Extraction of Vertical Profiles: Beam Broadening

At 15 km, ARMOR 3dB beam is 250 m tall

ARMOR vs XPR
- 12 stratiform events
- \( XPR_{ML} \) from Doppler velocity curvature
- Correlation: 0.97-0.98
- Bias:
  - \( ML_{TOP} = 166 \) m
  - \( ML_{BTM} = -70 \) m
ML Thickness vs Raindrop Size at Ground

$90\%$ of mean $D_m$ follows changes in $ML_{\text{thick}}$

$D_m = 0.372ML_{\text{thick}} + 1.187$, $ML$ in km
ML Height vs Raindrop Size at Ground

89% of mean $D_m$ follows changes in $ML_{\text{bottom}}$
ML impacts on raindrop size at ground

→ Raindrop size tends to be more affected by changes in thickness of ML than height
What about vertical evolution of the RSD?
Raindrop Size Distribution (RSD) Definitions and Background

- Modified Gamma Function (Testud et al. 2001):
  \[ N(D) = N_w f(\mu) \left( \frac{D}{D_m} \right)^\mu \exp \left[ -(4 + \mu) \frac{D}{D_m} \right] \]
  - \( D_m \) = Mass-weighted raindrop diameter
  - \( N_w \) = normalized intercept parameter (related to the number concentration of raindrops)
  - \( \mu \) = shape parameter of gamma distribution
  - \( D_m \) and \( N_w \) commonly used to describe RSD
  - Retrieval of shape parameter from radar is not very robust yet (Thurai and Bringi 2007)
Radar Retrieval of RSD Parameters

- 134,000 1-min RSD spectra from 2D-video disdrometer (2DVD) measurements in Huntsville and Iowa (IFloodS)
- T-matrix dual-pol radar scattering simulation:
  - C-band and S-band frequencies at daily mean temperature
  - Raindrop shape from bridge experiment (Thurai and Bringi 2005)
  - Gaussian canting angle distribution centered on 0° and σ=8°
  - DSD truncated at $D_{\text{max}}=3D_m$
  - 150 raindrops required in each spectra
- Prior to fitting, Gaussian noise added for radar measurement error
  - ARMOR: $\sigma(Z_h)=\pm1.2$ dB; $\sigma(Z_{DR})=\pm0.3$ dB
  - NPOL: $\sigma(Z_h)=\pm1.9$ dB; $\sigma(Z_{DR})=\pm0.4$ dB
- $D_m$ retrieved from polynomial function of $Z_{DR}$
- $N_w$ (number concentration) retrieved from power-law function of $Z_h$ and $D_m$
$D_m = 0.5969 + 1.7953Z_{DR} - 1.1111Z_{DR}^2 + 0.5171Z_{DR}^3 - 0.1360Z_{DR}^4 + 0.0142Z_{DR}^5$, for $-0.2 \leq Z_{DR} \leq 4.7 \text{dB}$

Relative uncertainty: 8% (12% for NPOL)
\[ N_w = 32.47Z_h D_m^{-7.102}, \text{ for } 0.25 \leq D_m \leq 2.0 \text{ mm} \]
\[ N_w = 93.60Z_h D_m^{-8.780}, \text{ for } 2.0 < D_m \leq 3.2 \text{ mm} \]
\[ N_w = 2.223Z_h D_m^{-5.641}, \text{ for } 3.2 < D_m \leq 6.0 \text{ mm} \]

Relative uncertainty: 18% (34% for NPOL)
Vertical Composite of RSD Retrievals

No apparent trend
Vertical Evolution of RSD as function of ML thickness

- Larger raindrops exited thicker ML
- $D_m$ varied less than 12%
- $D_m$ changed most for thinner MLs (fewer small raindrops)
- Generally more raindrops exited thicker ML
- Larger $N_w$ variability aloft
INSIDE THE ML: AGGREGATION or BREAKUP
FROM XPR measurements of particle flux

Aggregation became more dominate within ML as it grew thicker

Technique of (Drummond et al. 1996) to examine particle growth in ML

\[ \gamma = \frac{Z_{e,\text{snow}} V_{r,\text{snow}}}{Z_{e,\text{rain}} V_{r,\text{rain}}} \]

\( \gamma < 0.23 \): Aggregation dominates
  ➢ larger raindrops

\( \gamma > 0.23 \): Breakup dominates:
  ➢ smaller raindrops

Aggregation became more dominate within ML as it grew thicker
Q1: How do ML characteristics impact the physics of rainfall in the vertical column?

Hypothesis:
A thicker and lower ML results in larger raindrops.

1) Changes in ML thickness and height can describe 90% of the change in raindrop size
   • ML thickness tied to rate of snowflake melting (Mitra et al. 1990), which is dependent upon initial size and aggregation efficiency within the ML as well as RH conditions (Heymsfield et al. 2015)
   • Larger, rimed aggregates can contain more liquid than smaller individual ice crystals
     → Raindrop size tied to efficiency of snowflake aggregation

2) Vertical evolution of RSD similar beneath different MLs
   • Initial breakup of raindrops tends to dominate upon exiting the ML
   • Evaporation seems to dominate about 1 km below the ML
   • Further evaporation and breakup tends reduce size and concentration of raindrops as they approach the ground
Question 2

Can the ML be used to improve radar QPE?
ML and Rainfall Rate (R)

- Thicker ML tends to result in greater R (Cross-Correlation = 0.83)
- Examined 25 rainfall events and found good linear correlation (0.84)
  \[ R = 0.876 \exp(1.652ML_{thick}) \]
- However R(ML_{thick}) performed no better than Marshall-Palmer Z-R
Why did $R(ML_{\text{thick}})$ perform so poorly?
Too many variable that are rather difficult to measure (e.g., collisions, breakup, evaporation, etc.)

How about using $D_m$ instead?

However, single-parameter rainfall estimators are limited since $R$ has 3 unknowns:

$D_m, N_w$ and $\mu$ (also fall velocity)
Relationship between $R$, $Z$ and $D_m$

* $Z = aR^b$

* $R = \frac{10^{-4} \pi}{6} \int_0^\infty v(D)D^3 N(D) dD = F_R(\mu) N_w D_m^{4.67}$

* $Z = \int_0^\infty N(D) D^6 dD = F_Z(\mu) N_w D_m^{7}$

\[
\frac{Z}{R} = \frac{F_Z(\mu)}{F_R(\mu)} D_m^{2.33}
\]

∴ $a = \frac{F_Z(\mu)}{F_R(\mu)} D_m^{2.33}, b = 1$

Constant $F_Z(\mu)/F_R(\mu)$ ratio for $-2 < \mu \leq 15$
Empirical Fits to $R(Z_h)$ from 2DVD and T-matrix

Fits for each 0.1 mm wide bin of $D_m$

$Z = aR^b$

<table>
<thead>
<tr>
<th>$D_m$ [mm]</th>
<th>N</th>
<th>$a$</th>
<th>$b$</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5-0.6</td>
<td>02512 0058, 1.10</td>
<td>(0.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7-0.8</td>
<td>04968 0097, 1.03</td>
<td>(0.15)</td>
<td></td>
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<tr>
<td>0.9-1.0</td>
<td>08060 0161, 1.03</td>
<td>(0.16)</td>
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<tr>
<td>1.1-1.2</td>
<td>09268 0245, 1.03</td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.3-1.4</td>
<td>06602 0361, 1.03</td>
<td>(0.10)</td>
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<td></td>
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<tr>
<td>1.5-1.6</td>
<td>04594 0497, 1.04</td>
<td>(0.09)</td>
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<tr>
<td>1.7-1.8</td>
<td>02159 0655, 1.07</td>
<td>(0.10)</td>
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<td></td>
</tr>
</tbody>
</table>
Empirical Fit to Z-R coefficients stratified by $D_m$

- Empirically determined “a” coefficient within bounds of that predicted by theory (differed less than 12%)
- Power-law model of $v_T$ not accurate for all $D_m$ (Atlas et al. 1973)

Empirical coefficient of $R = AZ^B$

$A = 0.00650 D_m^{-2.186}$

Theoretical coefficient:

$A = F(\mu)^{-1} D_m^{-2.33}$
## Rainfall Estimators

<table>
<thead>
<tr>
<th>Estimator Type</th>
<th>Fitted Equation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>New: ( R(Z_h, D_m) )</td>
<td>( R = 6.50 \times 10^{-3} D_m^{-2.186} Z_h^{0.96} )</td>
</tr>
<tr>
<td>Traditional: Z-R</td>
<td>( R = 39.1 \times 10^{-3} Z_h^{0.61} ) \ OR ( Z = 210R^{1.66} )</td>
</tr>
<tr>
<td>Sharma et al. 2009: ( Z/D_m )-R</td>
<td>( R = 19.8 \times 10^{-3} \left( \frac{Z_h}{D_m} \right)^{0.741} ) \ OR ( \frac{Z}{D_m} = 199R^{1.35} )</td>
</tr>
</tbody>
</table>

* All fits for stratiform RSDs using 2DVD measurements in Huntsville and \( Z_h \) from radar scattering simulations.
Performance of Estimators: Relative to Disdrometer

<table>
<thead>
<tr>
<th>Type</th>
<th>FB</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(Z_h,D_m)</td>
<td>-3.3%</td>
<td>13%</td>
</tr>
<tr>
<td>Z-R</td>
<td>-0.9%</td>
<td>37%</td>
</tr>
<tr>
<td>Z/D_m-R</td>
<td>0.1%</td>
<td>28%</td>
</tr>
</tbody>
</table>

**Fractional Bias**

\[
FB = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{R_{est,i} - R_{2dvd,i}}{R_{2dvd}} \right),
\]

**Normalized Standard Error**

\[
NSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{R_{est,i} - R_{2dvd,i} - \overline{R_{est}} + \overline{R_{2dvd}}}{R_{2dvd}} \right)^2}
\]
Relative to Operational Rain Gauges
Single-Parameter Z-R: $Z = 210R^{1.66}$

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>0.84</td>
</tr>
<tr>
<td>Fractional Bias</td>
<td>-53.6%</td>
</tr>
<tr>
<td>Mean Absolute Error</td>
<td>57.2%</td>
</tr>
<tr>
<td>Normalized Standard Error</td>
<td>89.8%</td>
</tr>
</tbody>
</table>
Relative to Operational Rain Gauges

Dual-parameter Z-R: $Z/D_m$ - R

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>0.91</td>
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<tr>
<td>Fractional Bias</td>
<td>-44.8%</td>
</tr>
<tr>
<td>Mean Absolute Error</td>
<td>50.3%</td>
</tr>
<tr>
<td>Normalized Standard Error</td>
<td>78.5%</td>
</tr>
</tbody>
</table>
Relative to Operational Rain Gauges

New Type: \( R(Z_h, D_m) \)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>0.87</td>
</tr>
<tr>
<td>Fractional Bias</td>
<td>-39.3%</td>
</tr>
<tr>
<td>Mean Absolute Error</td>
<td>50.4%</td>
</tr>
<tr>
<td>Normalized Standard Error</td>
<td>68.1%</td>
</tr>
</tbody>
</table>
Spectral Performance of Estimators

As function of Rainfall

\[ Z/R \]

\[ Z/D_m - R \]

\[ R(Z_h, D_m) \]

\[ R(Z_h, Z_{DR}, K_{DP}) \]

Distance from Radar

Bias < 0

MAE

No Bias

Fractional Bias [% (solid)]

1-HR rainfall accumulation [mm]

Distance from Radar [km]
Radar QPE Range Related Biases

- Reflectivity bright-band causes positive bias of radar estimators at 120-150 km
  → $Z_e$ Bright Band

- What about the negative bias around 50-75 km from radar?
  → $Z_{DR}$ Bright Band

(from Giangrande and Ryzhkov 2008)
Consider radar beam intercepting different regions of the profile

For example:
- @ Distant range
- Due to Shielding

- Vertical profile of reflectivity (VPR) correction can give 20-50% improvement in R (Bellon et al. 2005)
- Can further improvement be gained by considering the vertical profile of polarimetric parameters (VP3)?
The $Z_{DR}$ bright band can negatively bias the radar QPE.

Can we devise a way to further correct this residual bias in $R(Z_h,Z_{DR})$?
Question 3

What are VP3 characteristics and how does they vary with rainfall intensity?
Largest vertical changes for stratiform occur around ML:
  • $Z_e$ and $Z_{DR}$ bright bands
  • $\rho_{hv}$ “dark” band outlines ML

Extracted ML related signatures 15 km from radar using over 2,500 minutes of ARMOR RHIs through stratiform
Thickness of Melting Signatures

- Adjusted dual-pol variables for beam broadening following Ryzhkov (2007) technique
  - Similar results as XPR comparison
- Agree with aircraft obs through ML (e.g., Stewart et al. 1984, Willis and Heymsfield 1989)
- $Z_e$ bright band thickest
  - Region of greatest concentration of large melting particles
- $\rho_{hv}$ dark band thinnest
  - Closely outlines region of melting hydrometeors
Characteristics of VP3

Focus on $Z_{DR}$ since $R(Z_{h}, Z_{DR})$

- Less than 0.2-0.3 dB variation of $Z_{DR}$ in rain
- In snow, $Z_{DR}$ can differ 1 dB from rain
- $Z_{DR}$ typically 1.1-2.9 dB in BB
- $Z_{DR}$ was 0.5-2.0 dB greater in BB than in rain ($Z_{e}$ BB intensity 3-10 dB)
- $Z_{DR}$ BB located below $Z_{e}$ BB
- $Z_{DR}$ BB thicker than ML but thinner than $Z_{e}$ BB
- $Z_{DR}$ BB intensity decreased with increasing rainfall intensity
- Lower $Z_{DR}$ in snow associated with less intense $Z_{DR}$ BB but greater $Z_{DR}$ in rain
Question 4

What are the impacts of VP3 variability on radar QPE?
Recall Idealized Example of $Z_{DR}$ negatively biasing $R(Z_h, Z_{DR})$

![Graph showing VPR correction applied to $R(Z_h, Z_{DR})$]

Bias due to VP3 25-55%
VP3 Correction for $Z_{DR}$
Simulated Operational Scan Strategy (NEXRAD VCP21: 9 angles every 6-min)

Time-height series of ARMOR $Z_{DR}$ on March 6, 2011

- RHI scans and beam model used to simulate PPI at different ranges from radar
- Simulated PPI at 30 km range
- Simulated PPI at 60 km
- BB DETECTED
Results of VP3 Correction to \( R(Z_h, Z_{DR}) \)

Rain Gauge:
- Accum: 13.1 mm

Radar+VP3:
- 11.4 mm

Radar+VPR:
- 9.2 mm
So what about other events?

<table>
<thead>
<tr>
<th>10 rainfall events</th>
<th>15-min Rain Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before VP3</td>
</tr>
<tr>
<td>Fractional Bias</td>
<td>-25%</td>
</tr>
<tr>
<td>Mean Absolute Error</td>
<td>37%</td>
</tr>
</tbody>
</table>

VP3 Correction Reduced Bias in 1-HR Rainfall Accumulation by 12%
Thicker and lower ML produces larger raindrops
  * Raindrop size tied to snowflake aggregation efficiency
Characteristics of ML itself are not sufficient to improve radar QPE except indirectly through a new type of estimator: \( R(Z_h, D_m) \)
  * Lower uncertainty than \( Z-R \) or \( Z/D_m-R \) type estimators
  * Comparable to combined dual-pol estimator at moderate rainfall intensity (i.e., \( R>3\text{mm/h} \))
  * Useful for rain rate retrieval from GPM DPR?
The $Z_e$ and $Z_{DR}$ bright bands are the most distinct features of VP3 profiles in stratiform precipitation. The $Z_{DR}$ bright band can bias dual-pol radar QPE as much as 50% bias. VPR correction to $R(Z_h,Z_{DR})$ can be detrimental. VP3 correction for $Z_{DR}$ bright band can reduce the relative bias of $R(Z_h,Z_{DR})$ by 9-12% and error by 6%.
Where to go from here?

- Response of sub-cloud thermodynamics to changes in ML characteristics and feedbacks with RSD evolution
  - Thicker ML $\rightarrow$ larger $D_m$ and greater $N_w$ $\rightarrow$ RH response?
- Sub-cloud microphysics: Collisions and Evaporation
  - Breakup at first, followed by evaporation, then both
- Can $R(Z_h, D_m)$ be useful to dual-freq. retrievals?
  - For example: GPM DPR
- Test VP3 correction on other dual-pol estimators
- Dynamic modification of VP3 based on relationships with $Z_e$ and $\rho_{hv}$
- What does a convective VP3 look like and is it detrimental to dual-pol radar QPE?
Thank You!

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My WIFE and kids
QUESTIONS?