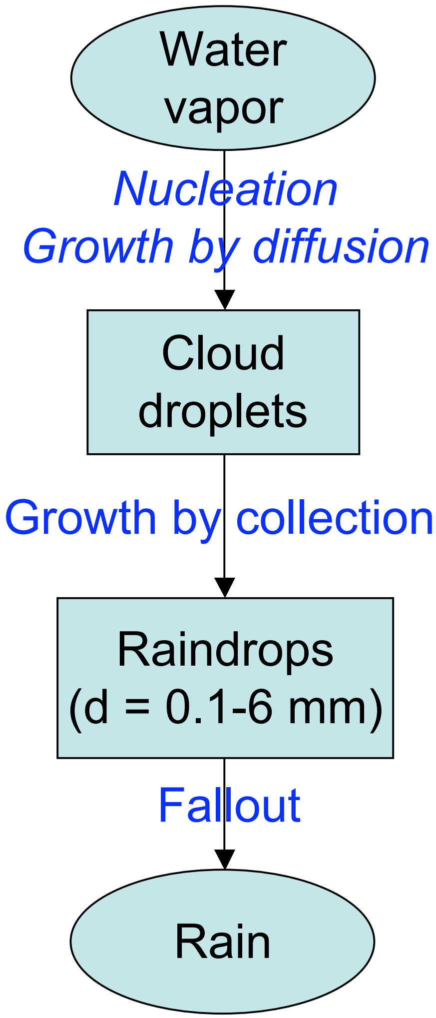
Formation and Growth of Ice Crystals (Ch. 9 of R&Y)

- The growth process:
  - Ice nucleation
  - Growth by diffusion (deposition)
  - Growth by collection
    - Riming
    - Aggregation
- Physics are complicated by the many types of ice particles
  - Pristine ice of various habits
  - Snow flakes
  - Aggregates (of snowflakes & other ice particles)
  - Graupel
  - Hail

A review of where we are on this topic

Representation of "paths to precipitation"

We considered in Chaps. 7-8:



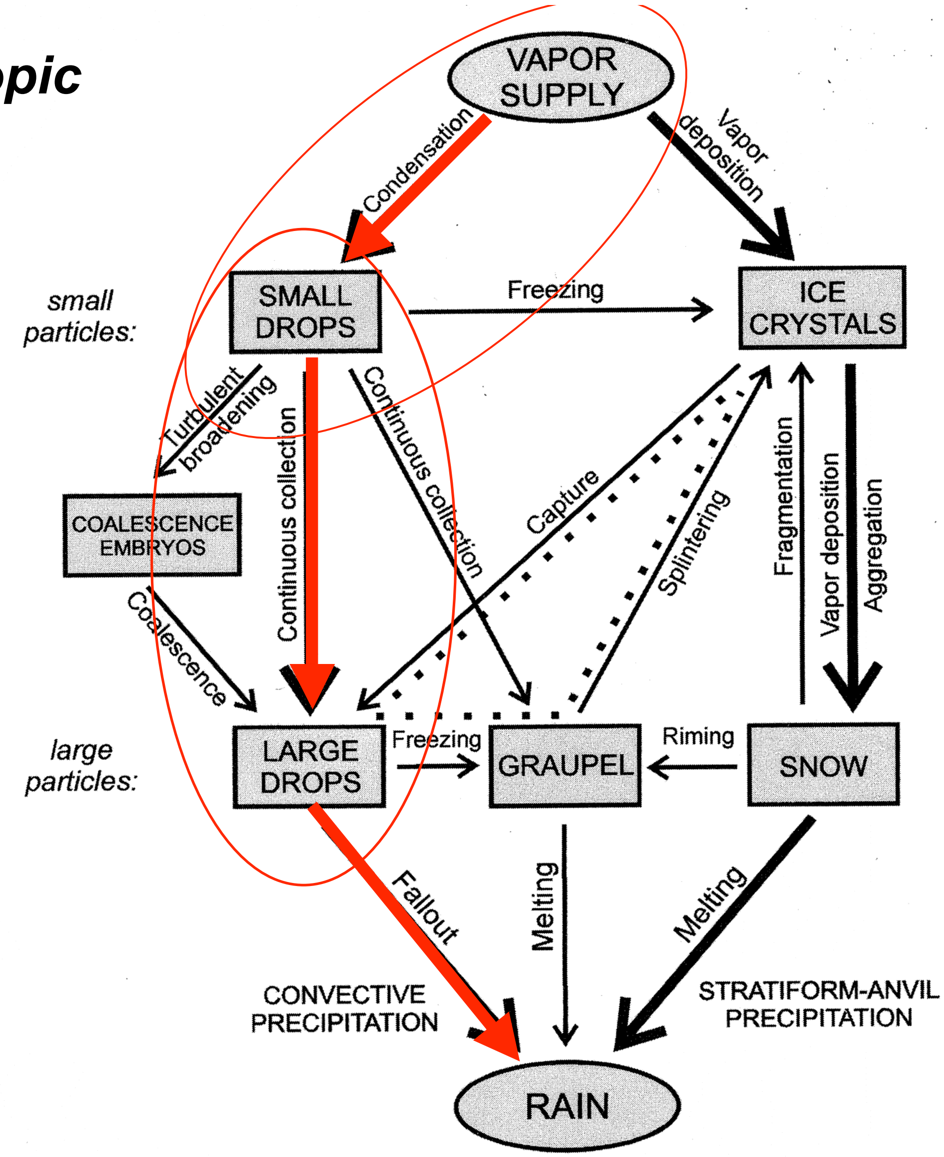
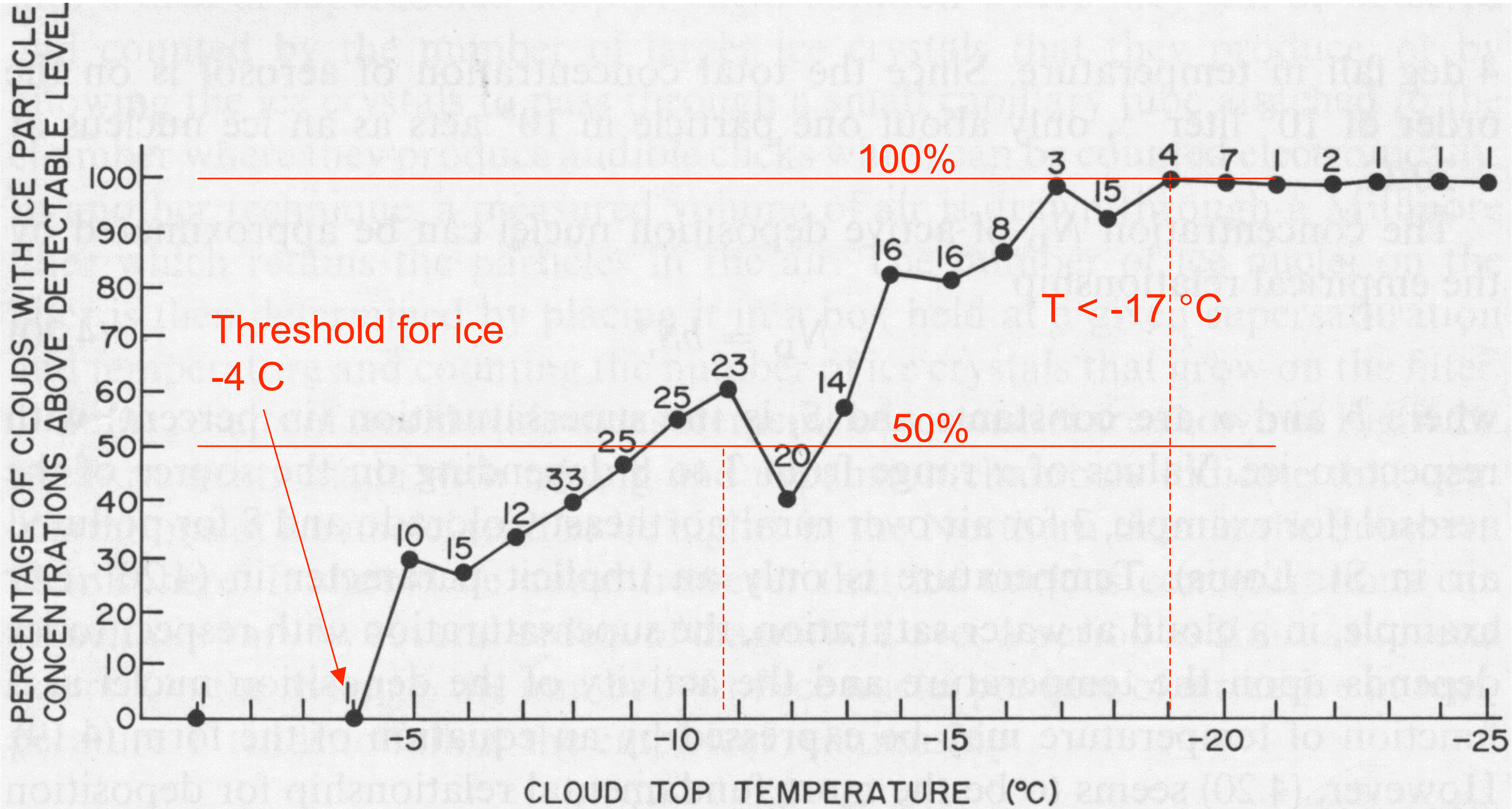


FIG. 8.10. A simplified schematic diagram of possible interactions between various categories of cloud particles. The bold arrows show the dominant pathways by which water vapor is transformed into rain via the "warm-rain" process (left-hand side) and via the "cold-rain" process (right-hand side) in heavy rain situations. The dotted set of arrows near the center of the diagram identifies a likely cyclical process for generating secondary ice particles via the rime-splintering mechanism of Hallett and Mossop (1974). The form of the diagram was adapted from Rutledge and Hobbs (1984).

Probability of ice in clouds (review)



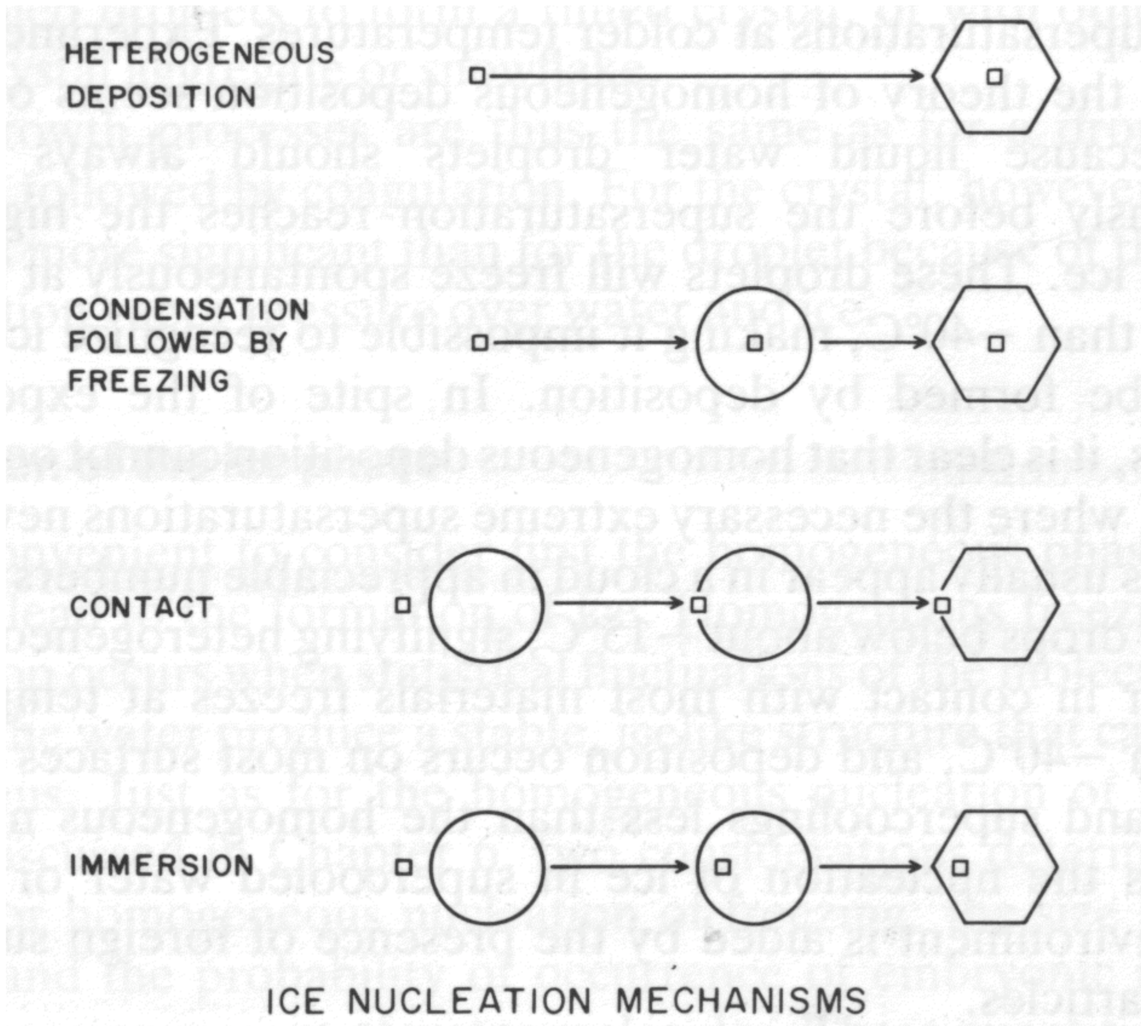
Percentage chance of ice being detected in clouds as a function of the cloud top temperature. Based on observations of 30 orographic cloud systems. Fig. taken from Wallace and Hobbs (1977).

Ice Nucleation
(4 types)

Homogeneous nucleation at
T = -40 °C.

Good ice nuclei have a surface or substrate that often has an ice-like crystalline structure.

Relative importance of the four mechanisms is not well established. However, see the following frame.



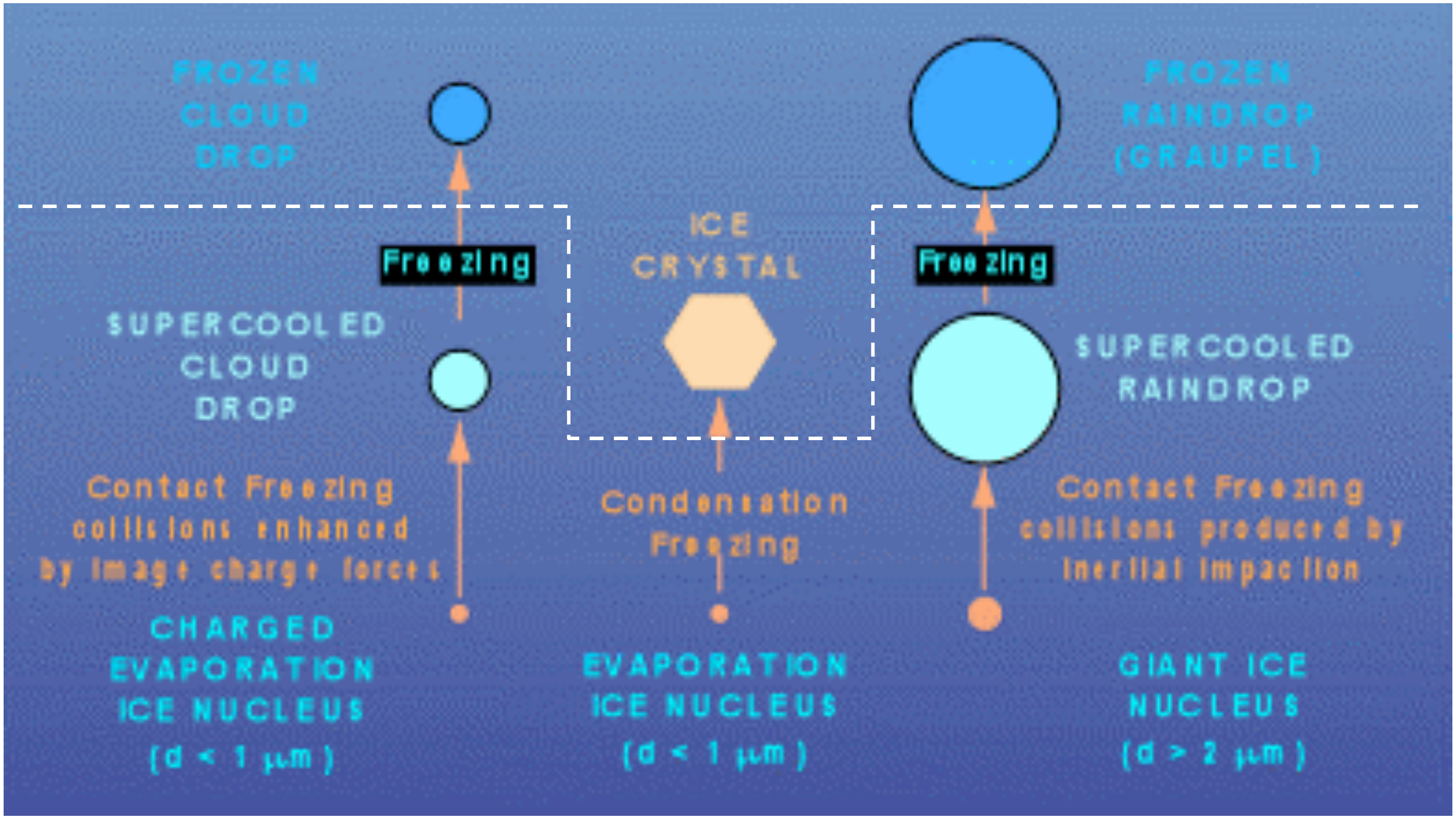
9.1. Schematic picture of the different ways atmospheric ice nu

Efficient pathways for ice nucleation in clouds

Ice particle produced

Nucleation process

Nucleus type



How does theory compare with observations?

Fletcher theory:

$$\ln N_{IN} = a(T_1 - T)$$

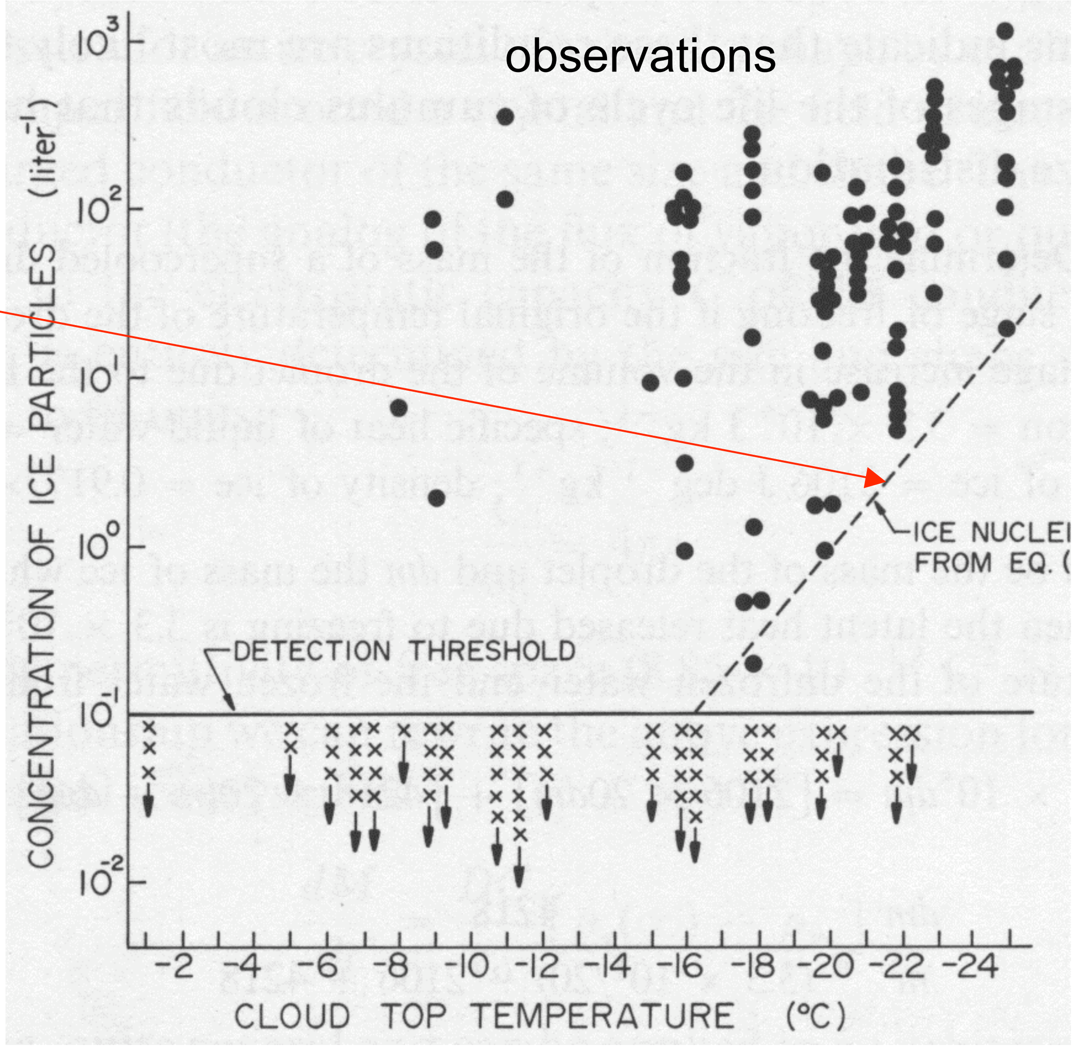
$$N_{IN} = \exp[a(T_1 - T)]$$

$$T_1 = -20\ °C$$

Concentrations of ice particles in clouds vs cloud top temperature. Taken from Wallace and Hobbs, Fig. 4.30.

Above relation states that N increases by 10 for every 4 °C decrease in temperature.

T (°C)	N_{IN} (per liter)
-20	1
-24	10
-28	100



Ice nuclei

Ice-forming temperatures of pure and natural substances.

Some combination of lattice matching and molecular binding, and low interfacial energy with ice accounts for the nucleating ability of a substance.

Not well explained by theory.

Effective natural nuclei:

- Kaolinite
- Bacteria?

Effective artificial nucleus:

- AgI (cloud seeding agent)

TABLE 9.1. *Temperatures at which different substances nucleate ice. (From Houghton, 1985)*

Substance	Crystal lattice dimension *a* axis (Å)	*c* axis (Å)	Temperature to nucleate ice (°C)	Comments
Pure substances				
Ice	4.52	7.36	0	—
AgI	4.58	7.49	−4	Insoluble
PbI_2	4.54	6.86	−6	Slightly soluble
CuS	3.80	16.43	−7	Insoluble
CuO	4.65	5.11	−7	Insoluble
HgI_2	4.36	12.34	−8	Insoluble
Ag_2S	4.20	9.50	−8	Insoluble
CdI_2	4.24	6.84	−12	Soluble
I_2	4.78	9.77	−12	Soluble
Minerals				
Vaterite	4.12	8.56	−7	
Kaolinite	5.16	7.38	−9	(Silicate)
Volcanic ash	—	—	−13	
Halloysite	5.16	10.1	−13	
Vermiculite	5.34	28.9	−15	
Cinnabar	4.14	9.49	−16	
Organic materials				
Testosterone	14.73	11.01	−2	
Chloresterol	14.0	37.8	−2	
Metaldehyde	—	—	−5	
β-Naphthol	8.09	17.8	−8.5	
Phloroglucinol	—	—	−9.4	
Bacterium *Pseudomonas Syringae*	—	—	−2.6	(Bacteria in leaf mold)

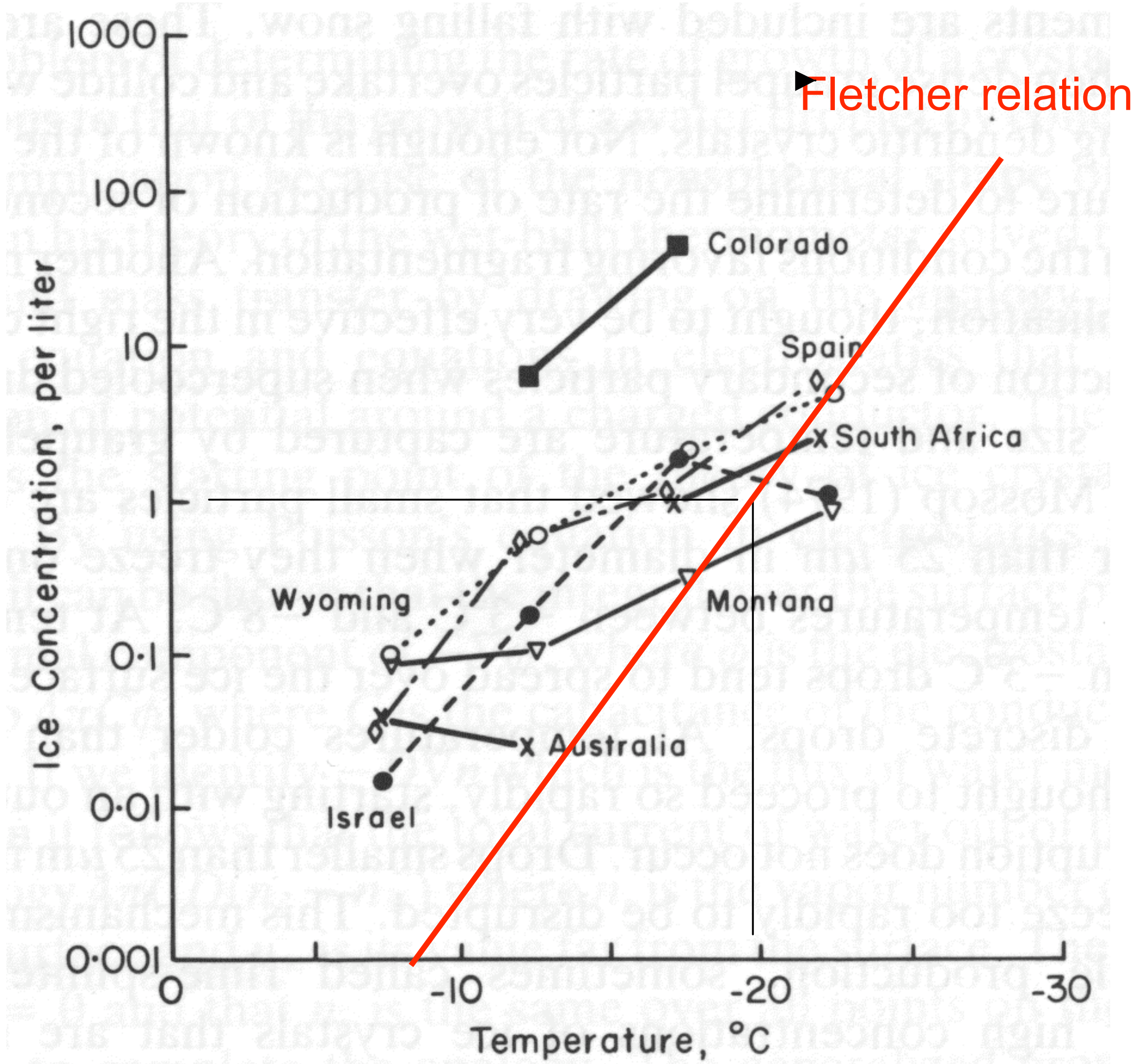
More observations on ice in clouds

Secondary ice production leads to greater ice concentration

Fig. 9.2. Summary of observed ice particle concentrations for clouds in which secondary processes of ice nucleation are believed to be unimportant.

Colder clouds have greater ice concentrations.

Measured concentrations exceed that predicted by the Fletcher theory (in many clouds)



Secondary ice production (ice multiplication) mechanisms:

- Fracture of ice (delicate) crystals
- Shattering or splintering of freezing raindrops
- *Hallett-Mossop ice multiplication mechanism* (rime splintering) based on observations.
- Hallet-Mossop criteria are:
  - Cloud droplet diameter > 25 μm
  - -8 °C < T < -3 °C
  - Presence of riming (implies that supercooled water and some ice are present)

Ice multiplication explains why the observed ice concentration far exceeds the Fletcher theoretical value

Fletcher theory:

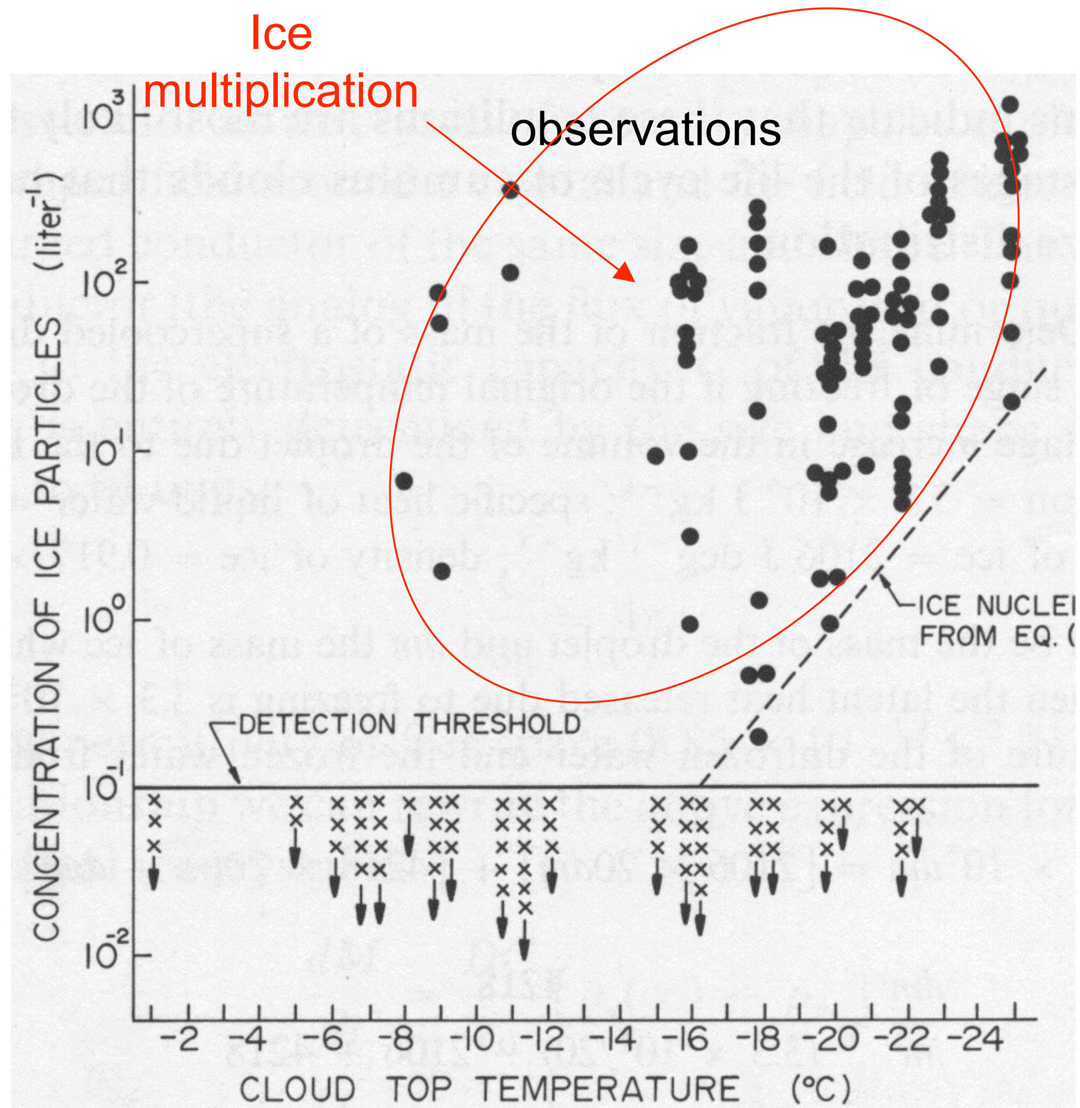
$$\ln N_{IN} = a(T_1 - T)$$

$$N_{IN} = \exp[a(T_1 - T)]$$

$T_1 = -20$ °C

Concentrations of ice particles in clouds vs cloud top temperature. Taken from Wallace and Hobbs, Fig. 4.30.

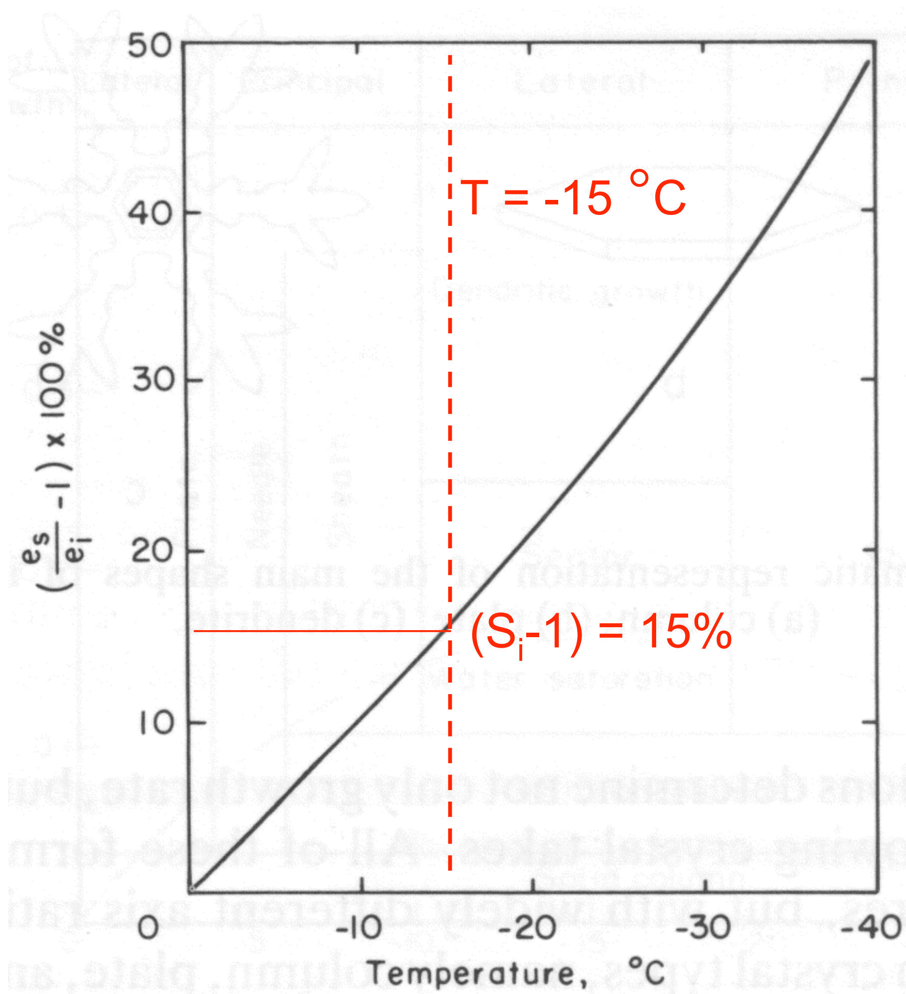
Above relation states that N increases by 10 for every 4 °C decrease in temperature.



Diffusional growth of ice crystals: similarity to diffusional growth of cloud droplets

- New definition of supersaturation w.r.t. ice:

  $S_i = e/e_i = (e/e_s)(e_s/e_i) = S(e_s/e_i)$
- $\therefore$ S_i can potentially attain high levels if the air is saturated w.r.t. water.



Supersaturation relative to ice of air at equilibrium saturation over water.

Growth equations

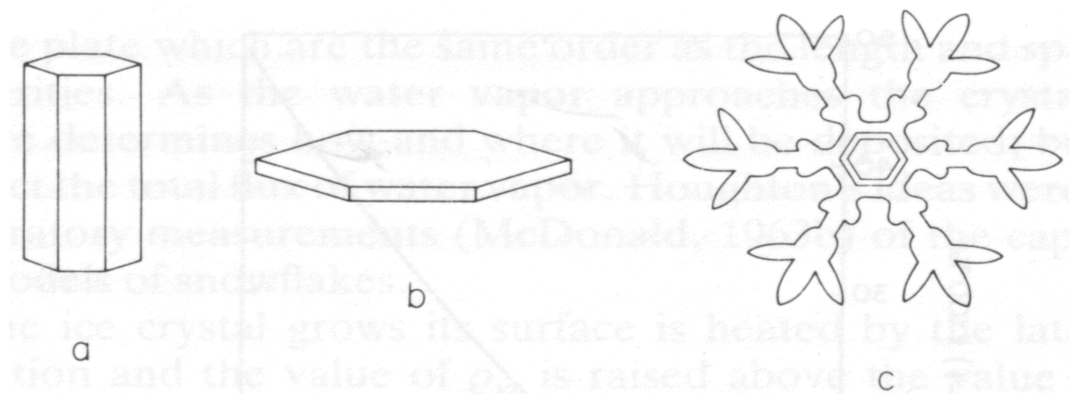
- Similarities and differences to condensational growth of water droplets.
- Mass rate of growth of diffusion:

$$dm/dt = 4\pi CD(\rho_v - \rho_{vr})$$

- Here C is the "electrical capacitance" (length units) that is a function of size and shape of the ice particle.
- Discuss the values of C for different crystal habits.
- This forces us to divert to a discussion of *crystal habits*.

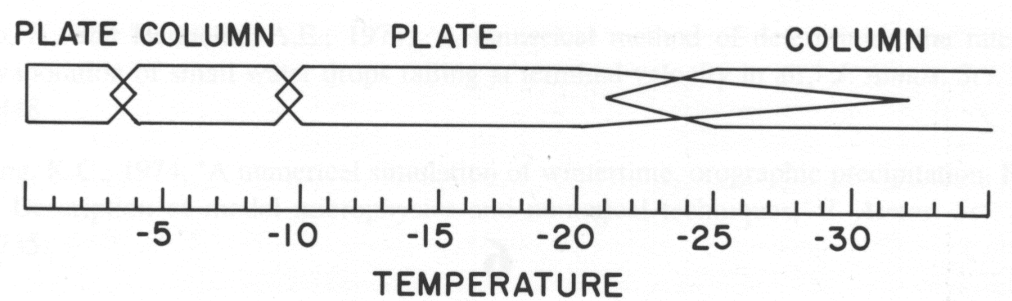
Crystal habits

- Dependence on T and S_i
- Basic types of ice crystals: column, plate, dendrite



T (C)	*Habit*	*Types of crystal at slight water supersaturation*
0 to -4	platelike	thin hexagonal plates
-4 to -10	prismlike	Needles (-4 to -6 C)
		Hollow colums (-5 to -10)
-10,-22	platelike	sector plates (-10 to -12)
		dendrites (-12 to -16)
		sector plates (-16 to -22)
-22, -50	prismlike	hollow columns

Temperature dependence only



Added complexities from ice supersaturation, S_i

(Temperature and S_i-1 dependence)

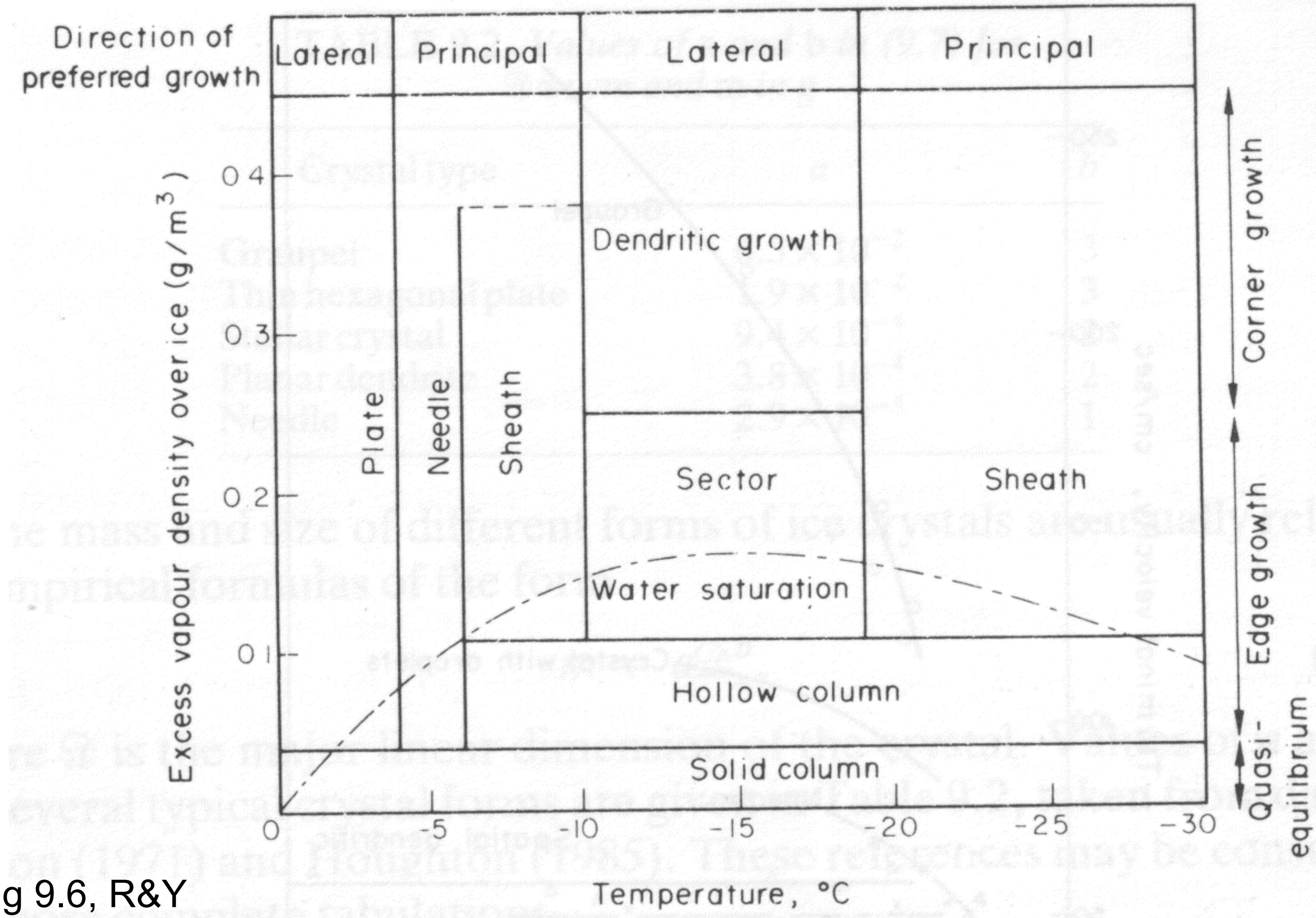
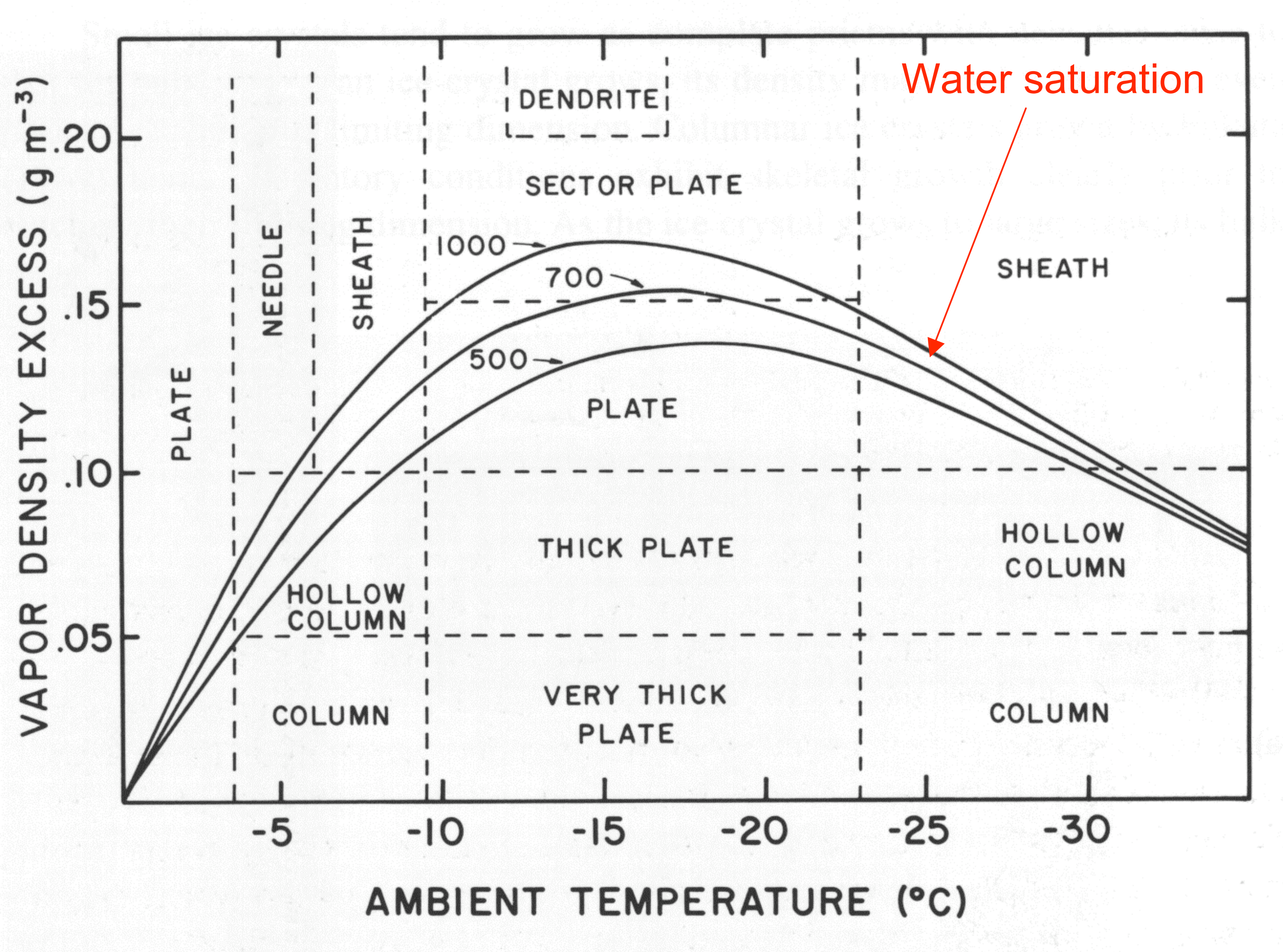
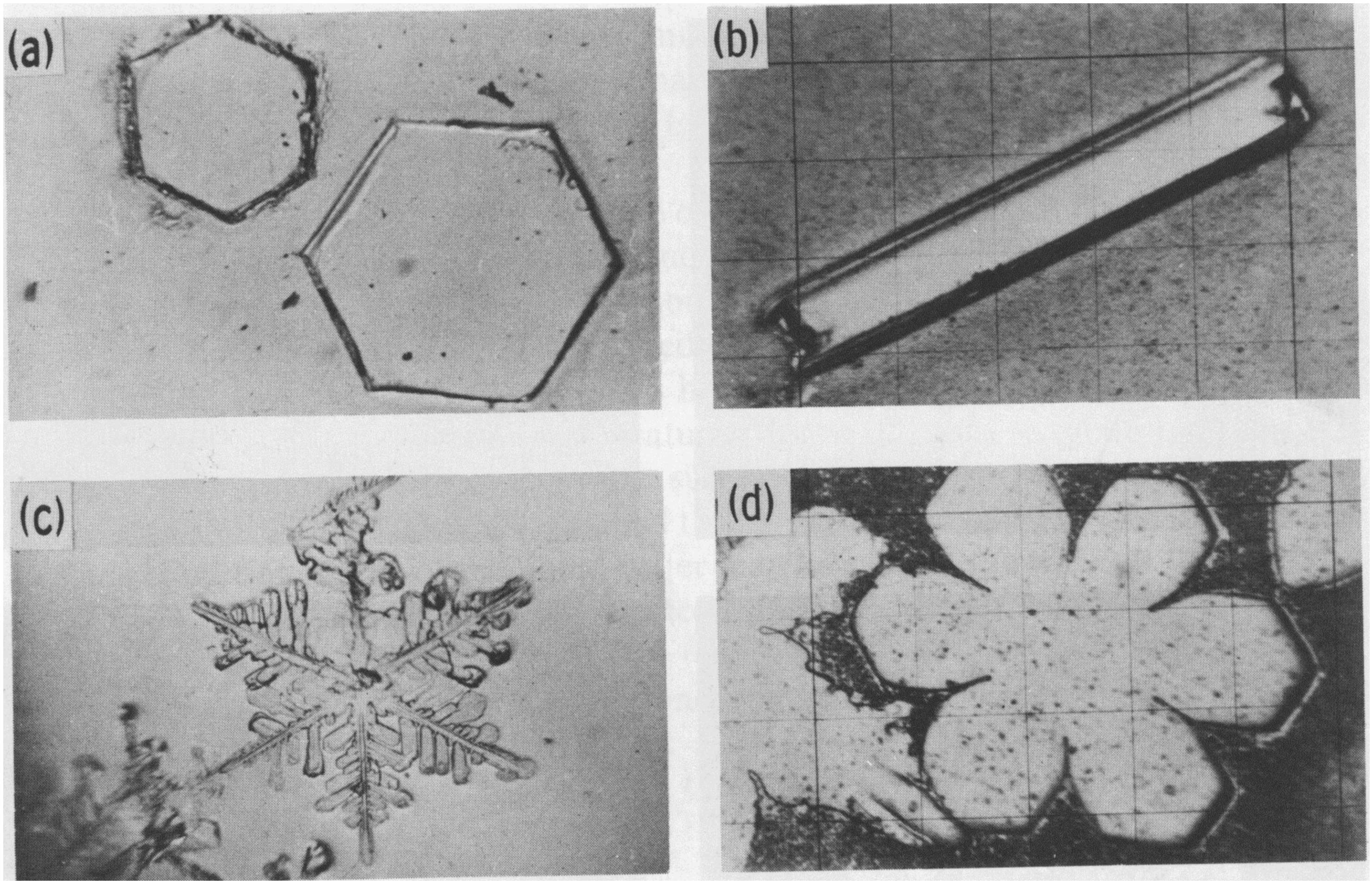


Fig 9.6, R&Y



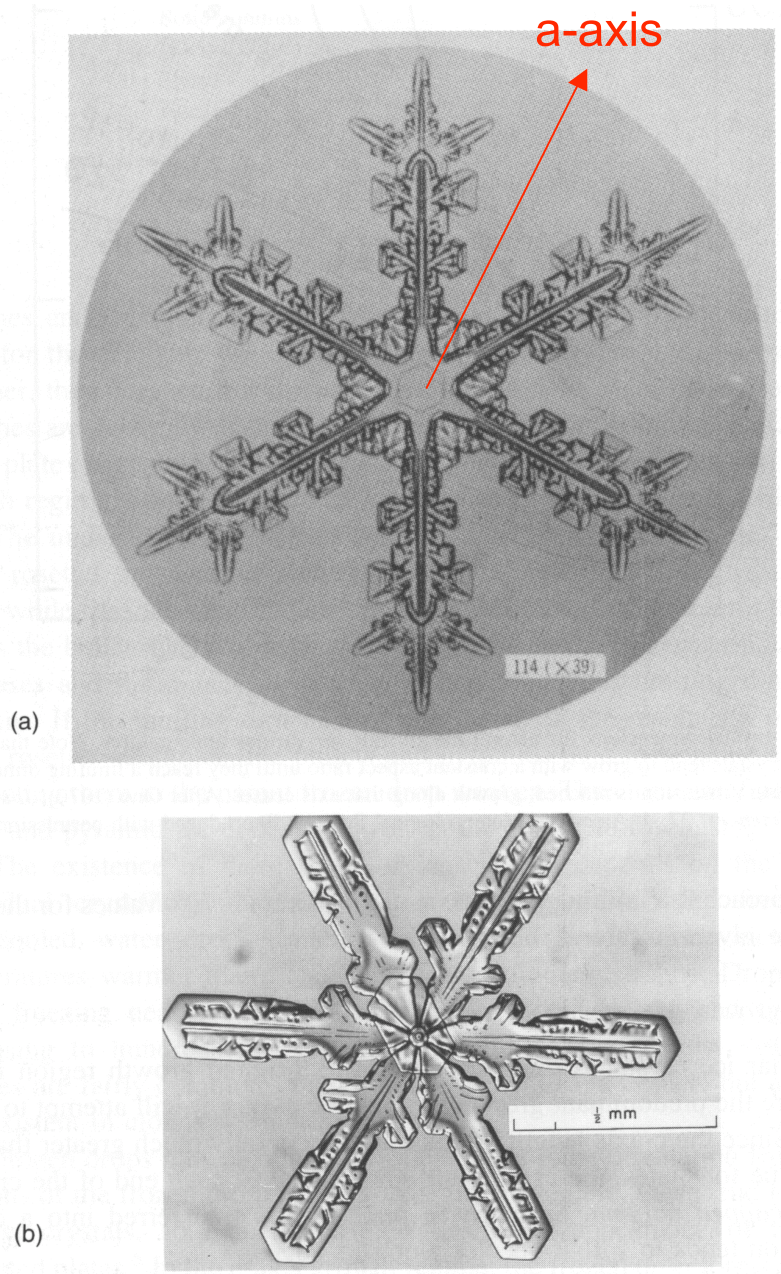
Simple crystal habits as a function of T and S_i. These represent habits attained by large ice crystals. The vapor density excess for water saturation at 1000, 700, and 500 mb are shown to illustrate the variation with altitude. Taken from Fig. 6.2 of Young (1993).

Examples of ice crystal habits



Examples of ice crystals which have grown from the vapor phase: (a) hexagonal plates, (b) column, (c) dendrite, and (d) sector plate. Taken from Fig. 4.32 of Wallace and Hobbs (1977).

Examples of a-axis growth regime ice crystals (left), and c-axis growth regime crystals (right and below). Taken from Figs. 6.3 and 6.4 of Young (1993).



6.4 Examples of a-axis growth regime ice crystals. (a) Dendrite (P1c). (b) sector p

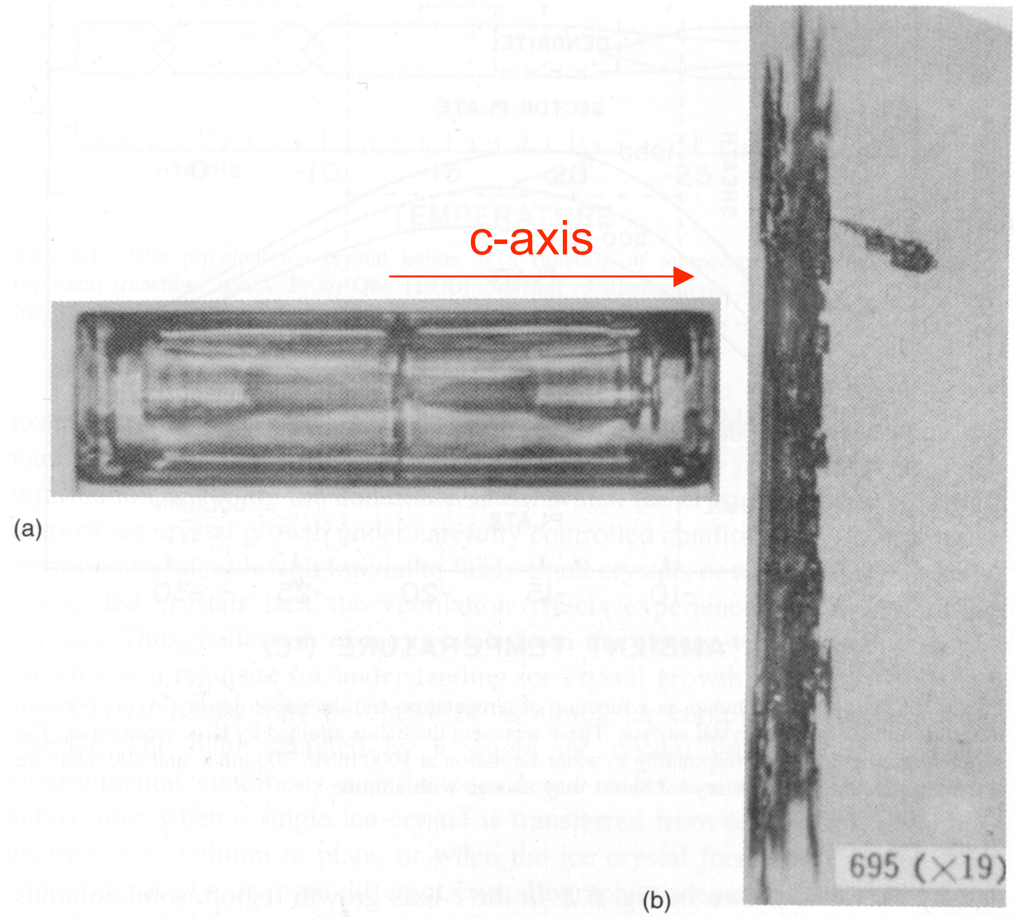


FIG. 6.3. Examples of c-axis growth regime ice crystals. (a) Banded column (C1e); (b) needle

Excellent information and pictures at this site

- http://www.its.caltech.edu/~atomic/snowcrystals/class/class.htm

I expect you to study the information at this site!

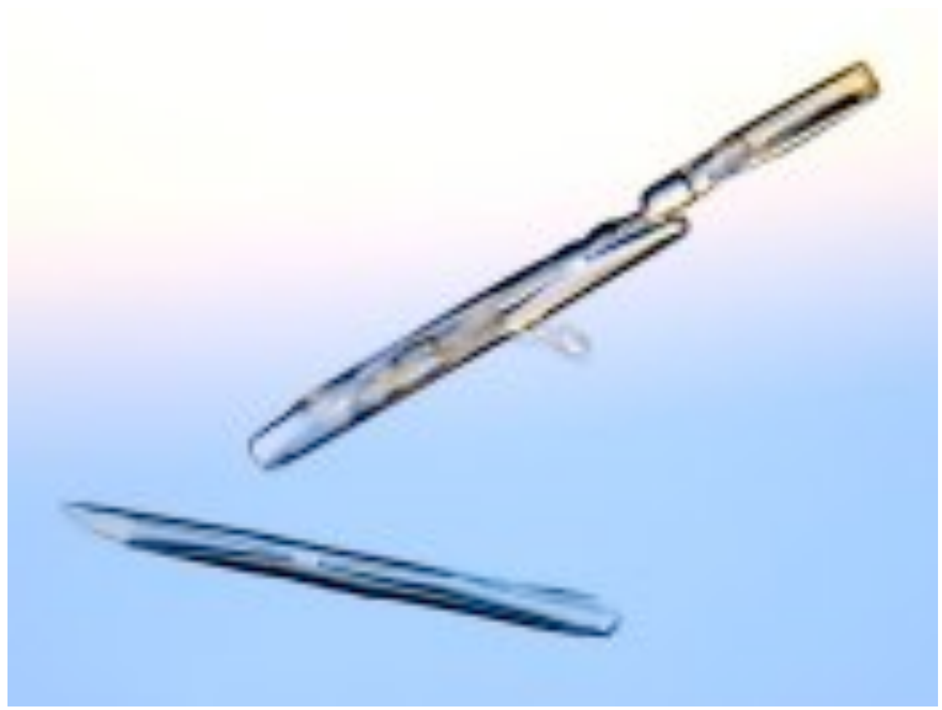
Stellar dendrites

Role in:

a) Ice multiplication via ice crystal fracture
b) Aggregation

Sectored plates

Needles

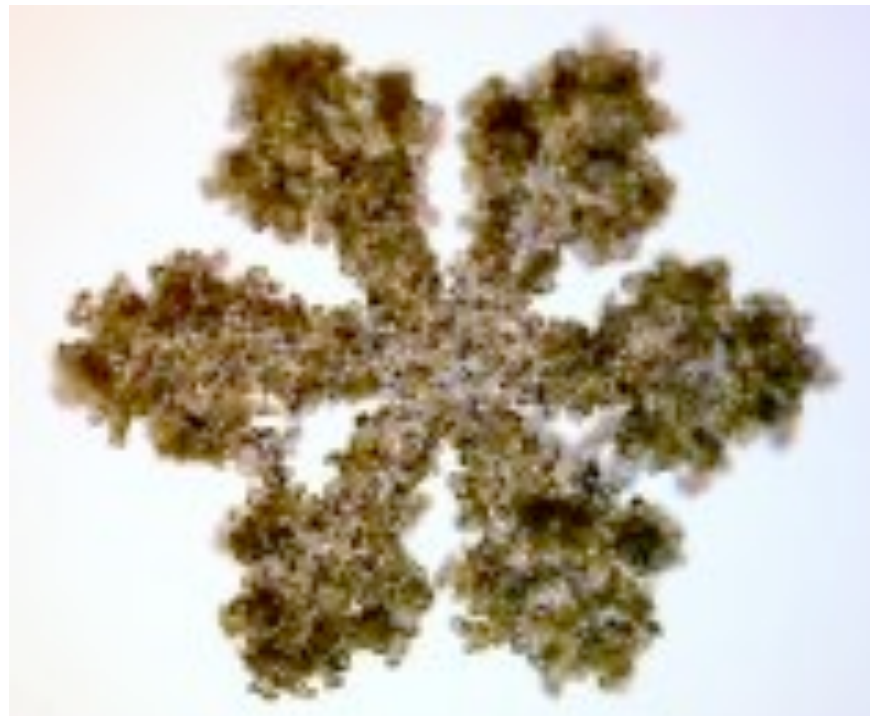


Spatial dendrites

Capped columns

Rimed crystals
(presence of supercooled cloud droplets)



More on growth equations

Physics of diffusion of vapor and heat:

$$\frac{dm}{dt} = 4\pi CD(\rho_\infty - \rho_{s,x})$$

$$\frac{dQ}{dt} = 4\pi CK(T_\infty - T_x)$$

Electrostatic analogy

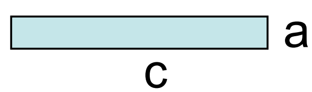
$$I = 4\pi C\Lambda(V_\infty - V_0)$$

I – electric current

C – capacitance

Λ – conductivity

V – potential



Values of capacitance

Oblate spheroid

$$C = \frac{ae}{\sin^{-1} e}$$

$$e = \left(1 - \frac{c^2}{a^2}\right)^{1/2}$$

Thin plates

$$C = \frac{2a}{\pi}$$

Columns

$$C = \frac{ce}{\ln\left[\frac{(1+e)c}{e}\right]}$$

$$e = \left(1 - \frac{a^2}{c^2}\right)^{1/2}$$

Needles (c>>a)

$$C = \frac{c}{\ln\left(\frac{2c}{a}\right)}$$

Integration of the growth equation (9.2, R&Y)*

Mass of a crystal with bulk density ρ_x: $m = 4\sqrt{3}a^2 c\rho_x = 4\sqrt{3}a^3\Gamma\rho_x \quad (\Gamma = c/a)$

For a constant aspect ratio Γ the differential of the above is: $dm = 12\sqrt{3}\rho_x \Gamma a^2 da$

Assuming a thin plate (using C for a thin plate): $dm = 8aD(\rho_\infty - \rho_{s,x})dt$

Equating the previous two expressions yields: $a = \left[a_0 + \frac{4D(\rho_\infty - \rho_{s,x})}{3\sqrt{3}\Gamma\rho_x}t\right]$

Assuming a very small initial crystal: $m = 4.6794\left[\frac{\left[D(\rho_\infty - \rho_{s,x})\right]^3}{\Gamma\rho_x}\right]^{1/2} t^{3/2} \sim t^{3/2}$

In the above, growth rate is large for small bulk density ρ_x and aspect ratio Γ

Examples for plate growing at constant thickness, and needle growing at constant diameter

*Taken from Young (1993), Ch. 6

Now we consider a plate crystal expanding at constant thickness

$$dm = 8\sqrt{3}ac\rho_x da = 8aD(\rho_\infty - \rho_{s,x})dt$$

Integration from an initial size a_0 gives

$$a = a_0 + \frac{D(\rho_\infty - \rho_{s,x})}{\sqrt{3}C\rho_x}t$$

For very small crystal sizes, the mass as a function of time is

$$m = \frac{4D^2(\rho_\infty - \rho_{s,x})^2}{\sqrt{3}C_*\rho_x}t^2 \sim t^2$$

Thus, a crystal growing at constant thickness grows at a faster rate (t^2) than one growing at constant aspect ratio ($t^{3/2}$)

Eq. (9.4): Growth equation

$$\frac{dM}{dt} = \frac{4\pi C(S_i - 1)}{\left(\frac{L_s}{R_v T} - 1\right)\frac{L_s}{KT} + \frac{R_v T}{e_{si}(T)D}}$$

Same physics as in Eq. (7.17)

Simultaneous treatment of diffusion of vapor onto crystal, and diffusion of heat (due to sublimation) away from cyrstal surface

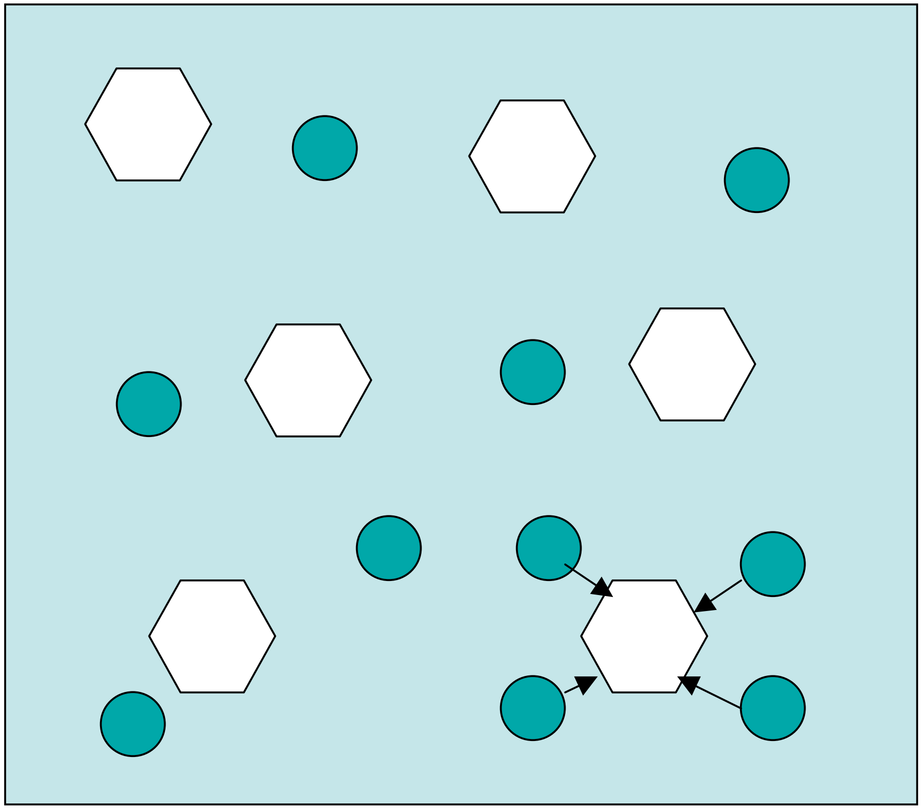
Kinetic effects are not included. Thus (9.4) will lead to an overestimate

Normalized growth rate (Fig. 9.4) is determined from the following expression:

$$\frac{dM/dt}{4\pi C} = \frac{(S_i - 1)}{\left(\frac{L_s}{R_v T} - 1\right)\frac{L_s}{KT} + \frac{R_v T}{e_{si}(T)D}}$$

Bergeron process

When ice crystals and (supercooled) water droplets coexist, the ice crystals grow (by deposition) while the water droplets evaporate.

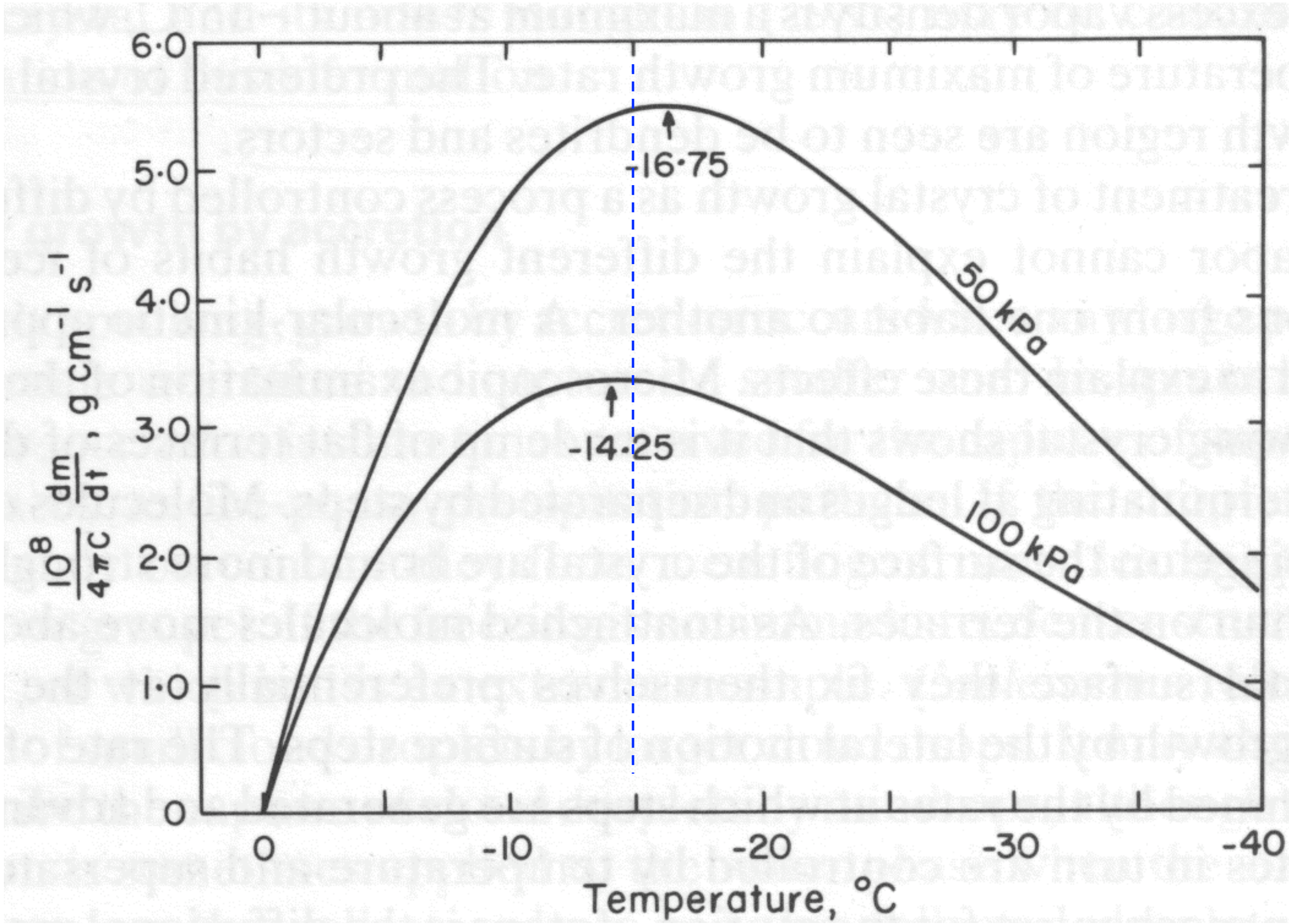


Why?

The equilibrium saturation vapor pressure over ice is less than that over water.

For continued growth, water droplets need to be re-nucleated or their size must be maintained by condensational growth, which implies (S-1) > 0 (and S_i-1 is large!).

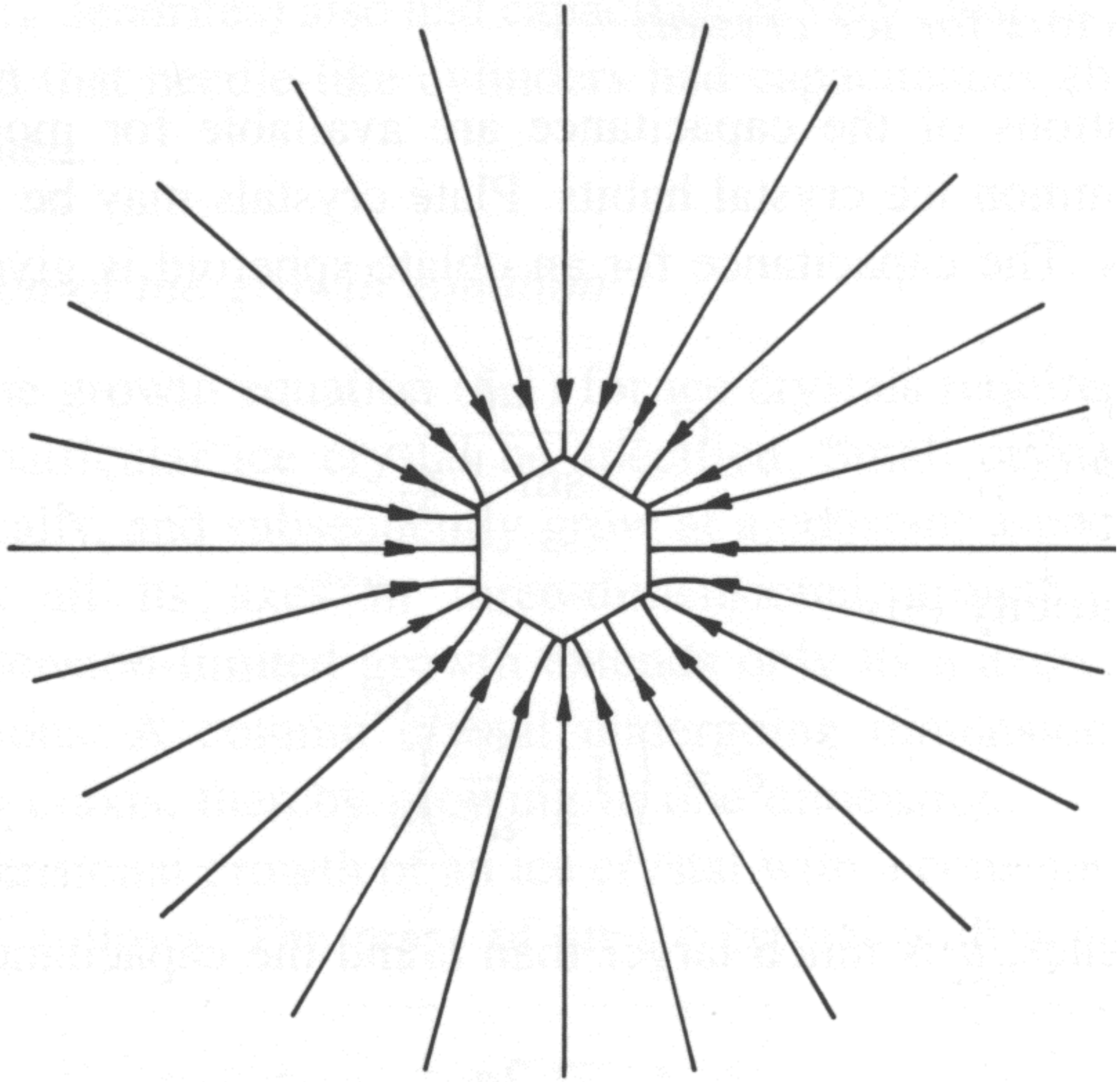
Results



Normalized ice crystal growth rate as a function of temperature.
(Fig. 9.4 from RY)

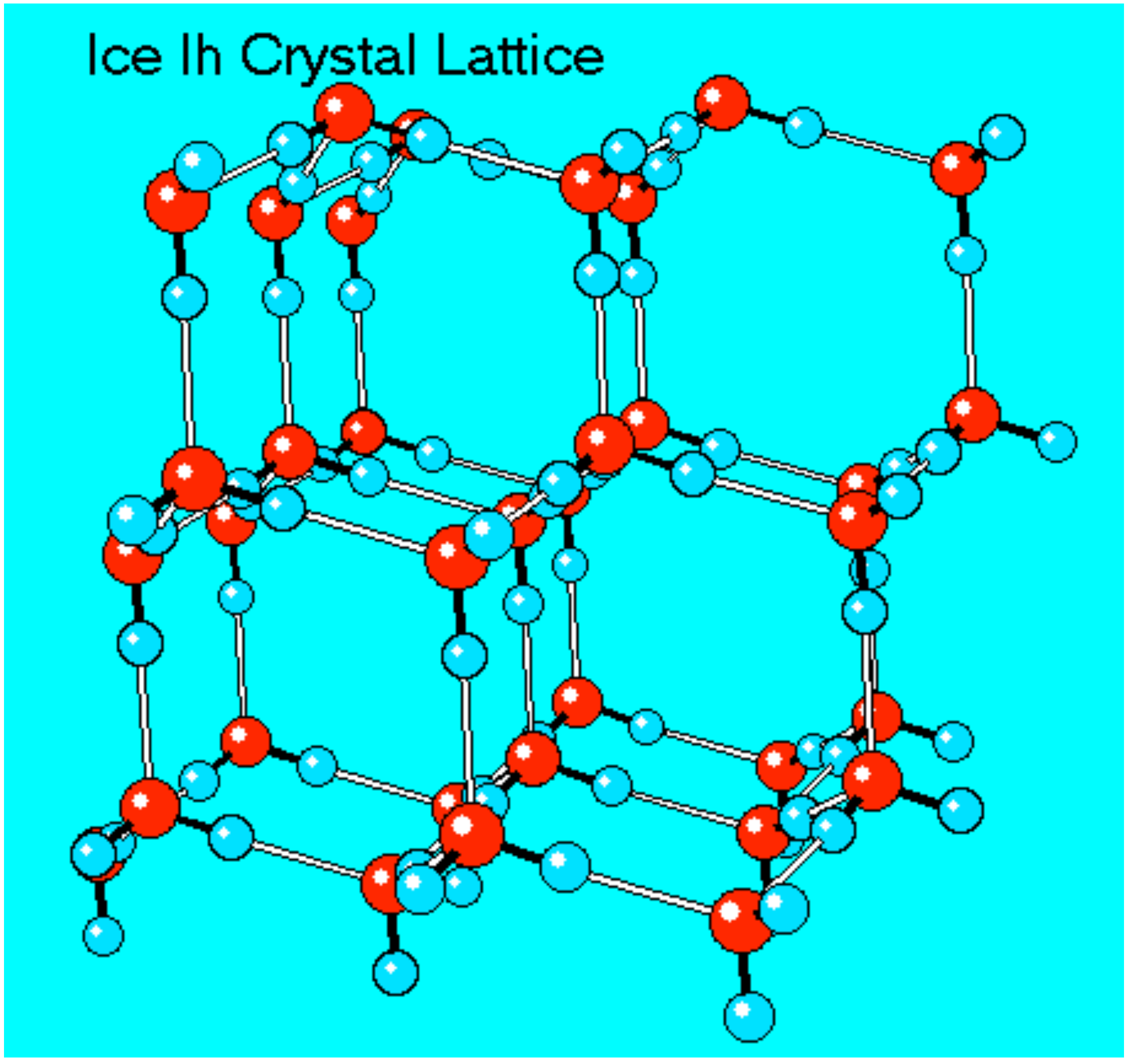
A maximum in growth rate (dm/dt) near -15 °C

But, the growth on the ice crystal is not symmetric as shown below



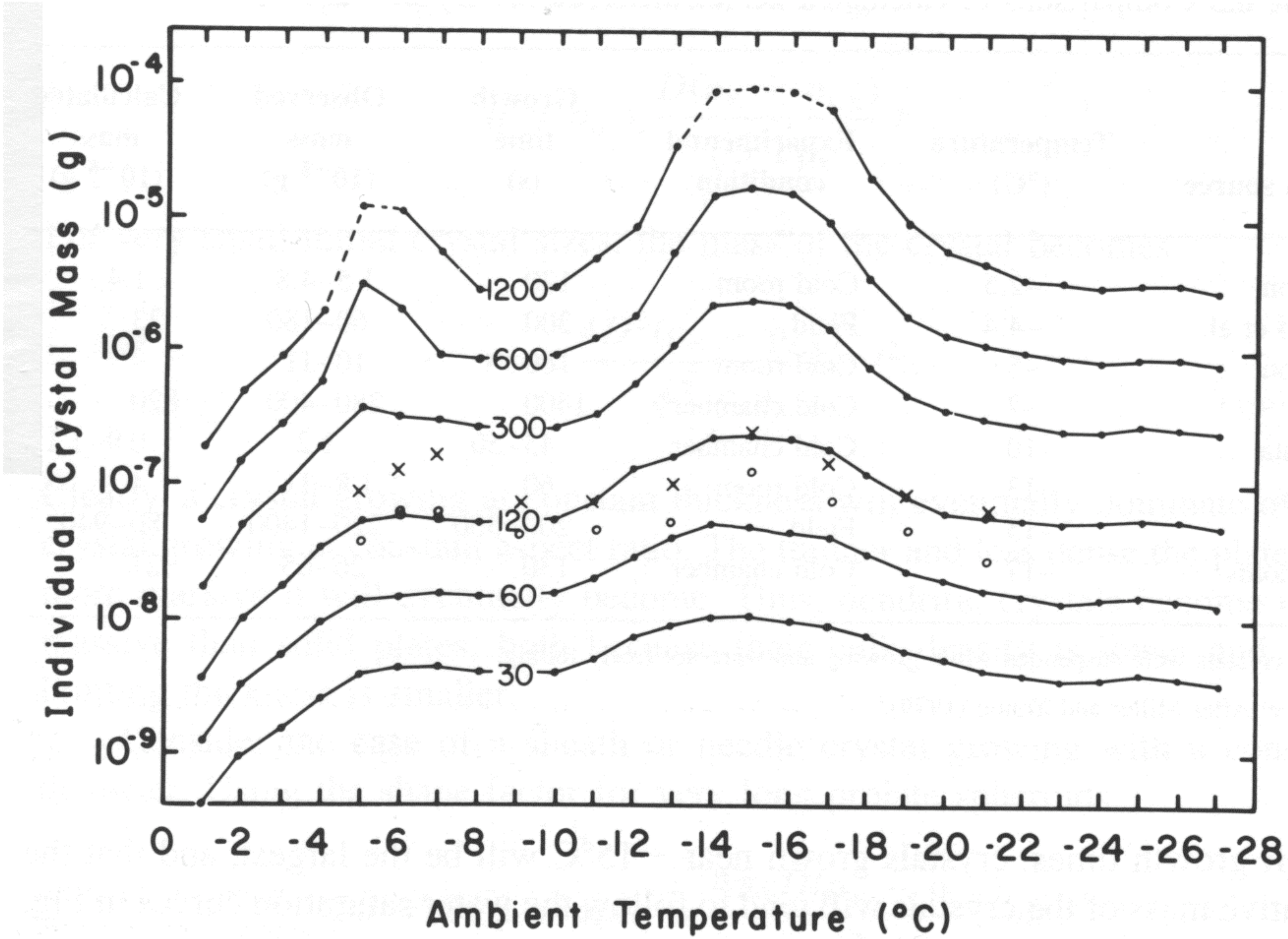
Flux lines around a plate crystal in two dimensions (based on electrolytic tank experiments). Fig. 6.8 from Young (1993)

Aside: Ice structure

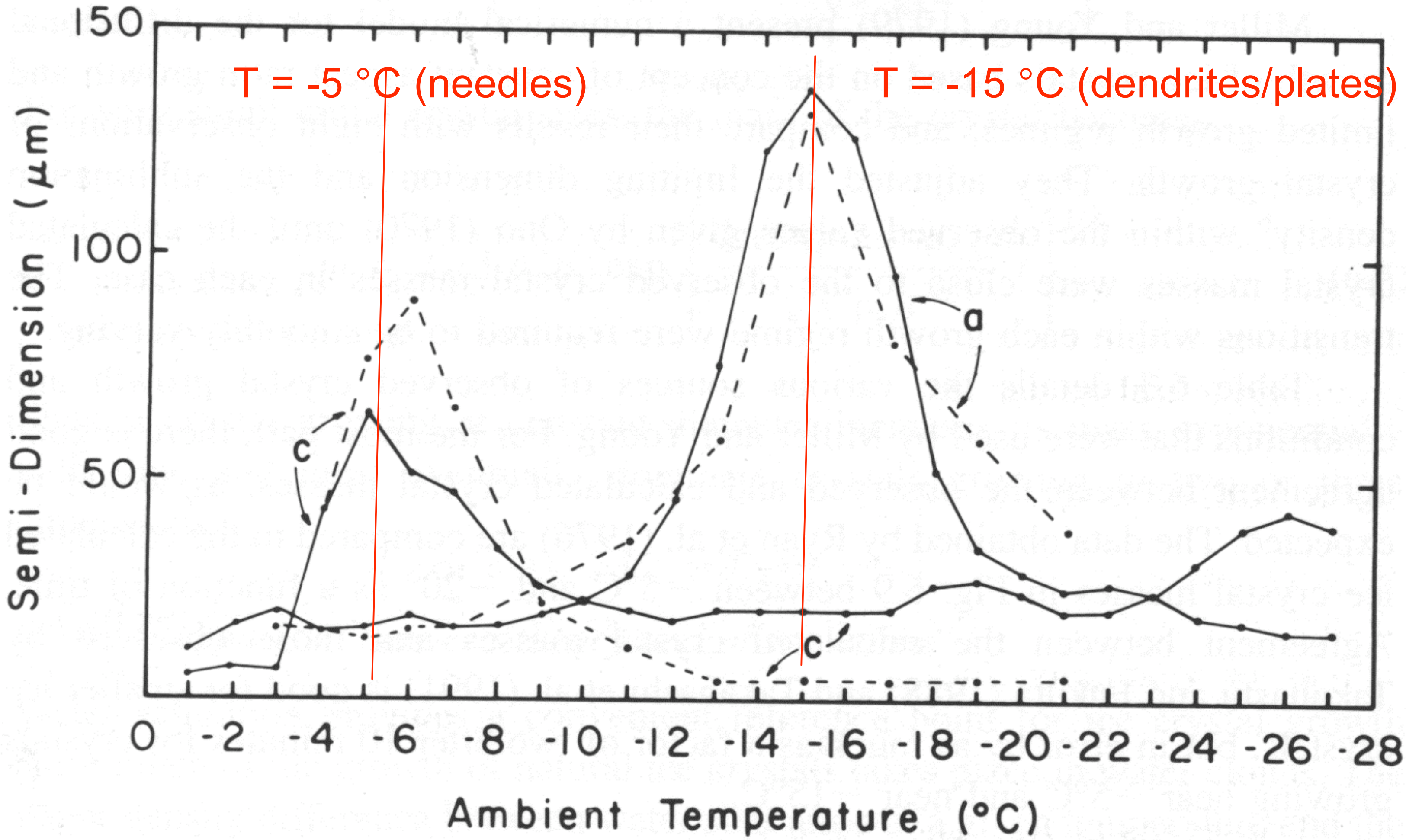


http://www.its.caltech.edu/~atomic/snowcrystals/ice/ice.htm

Some calculations



Predicted crystal mass after times annotated of growth at water saturation and constant ambient temperature. Fig. 6.9 of Young (1993)



Predicted crystal semi-dimensions after 120 s of growth at water saturation. Fig. 6.10 from Young (1993)

Table 6.3 Comparisons of calculated versus observed ice crystal mass.

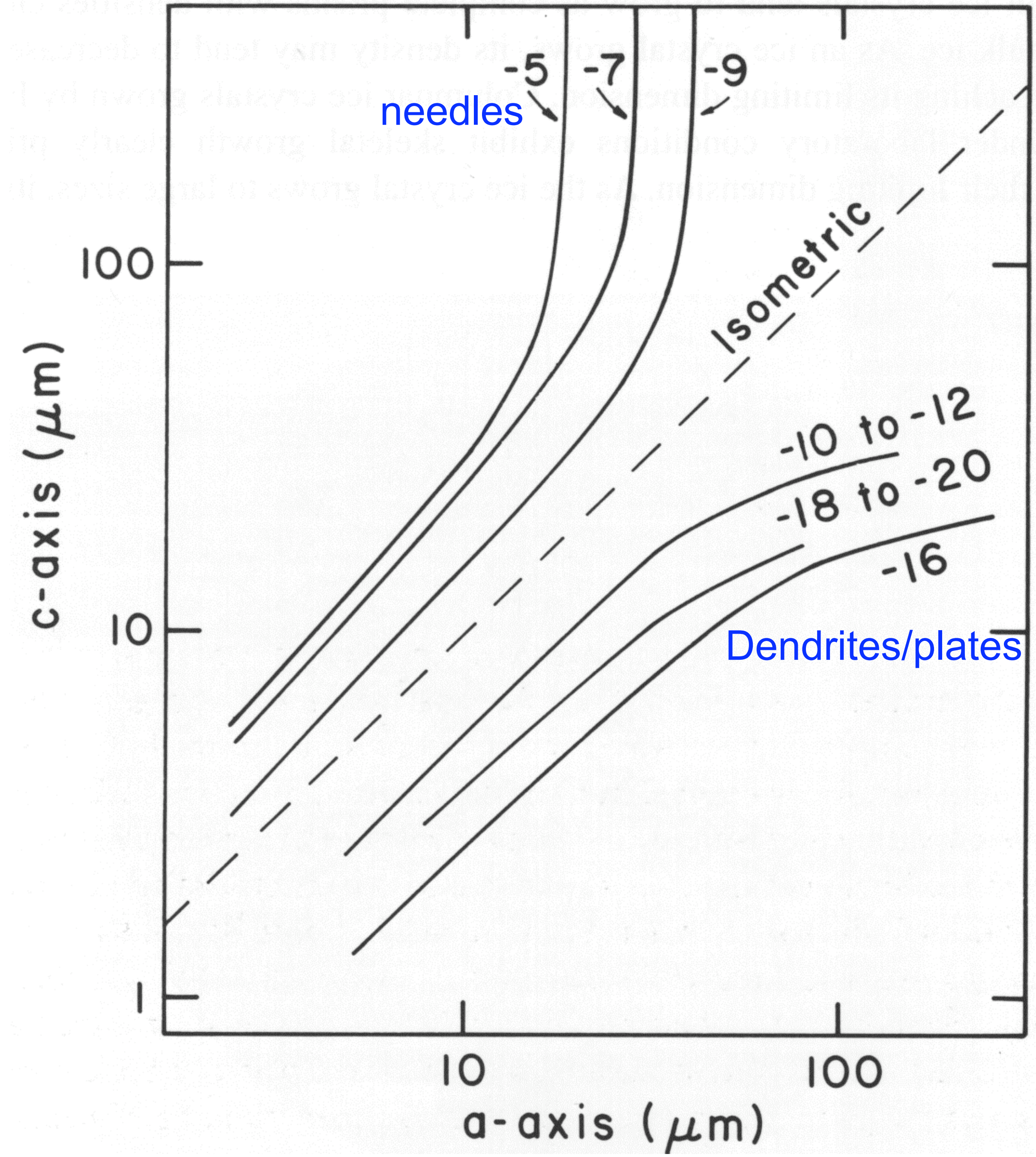
Data source	Temperature (°C)	Experimental condition	Growth time (s)	Observed mass (10^{-8} g)	Calculated mass (10^{-8} g)
Mason	–2.5	Cold room	120	1.5–4.8	1.4
Isono et al.	–4.4	Field	300	60–180	23
Mason	–5	Cold room	160	10–11	9
Ono	–7	Cold chamber[a]	1400	280–400	820
Fukuta	–10	Cold chamber	45–50	1.2	0.9–1.1
Shaw	–13	Cold room	60	1.8–11	4
Ono	–15	Field	200–500	220–1400	80–940
Reynolds	–17	Cold chamber	130	26–65	21

[a] Ice crystals were suspended while growing and were not freely falling.

Source: After Miller and Young (1979).

The ventilation factor does play a role in ice cyrstal growth

Ice crystal dimensions for natural ice crystals for various temperatures. Note that in all cases, the ice crystals tend to grow with a constant aspect ratio until they reach a limiting dimension (several tens of microns). [Fig. 6.5 from Young 1993, after Ono 1970).



Ice density

Important for:

a) Terminal fall speed relations

b) Interpretation of radar reflectivity

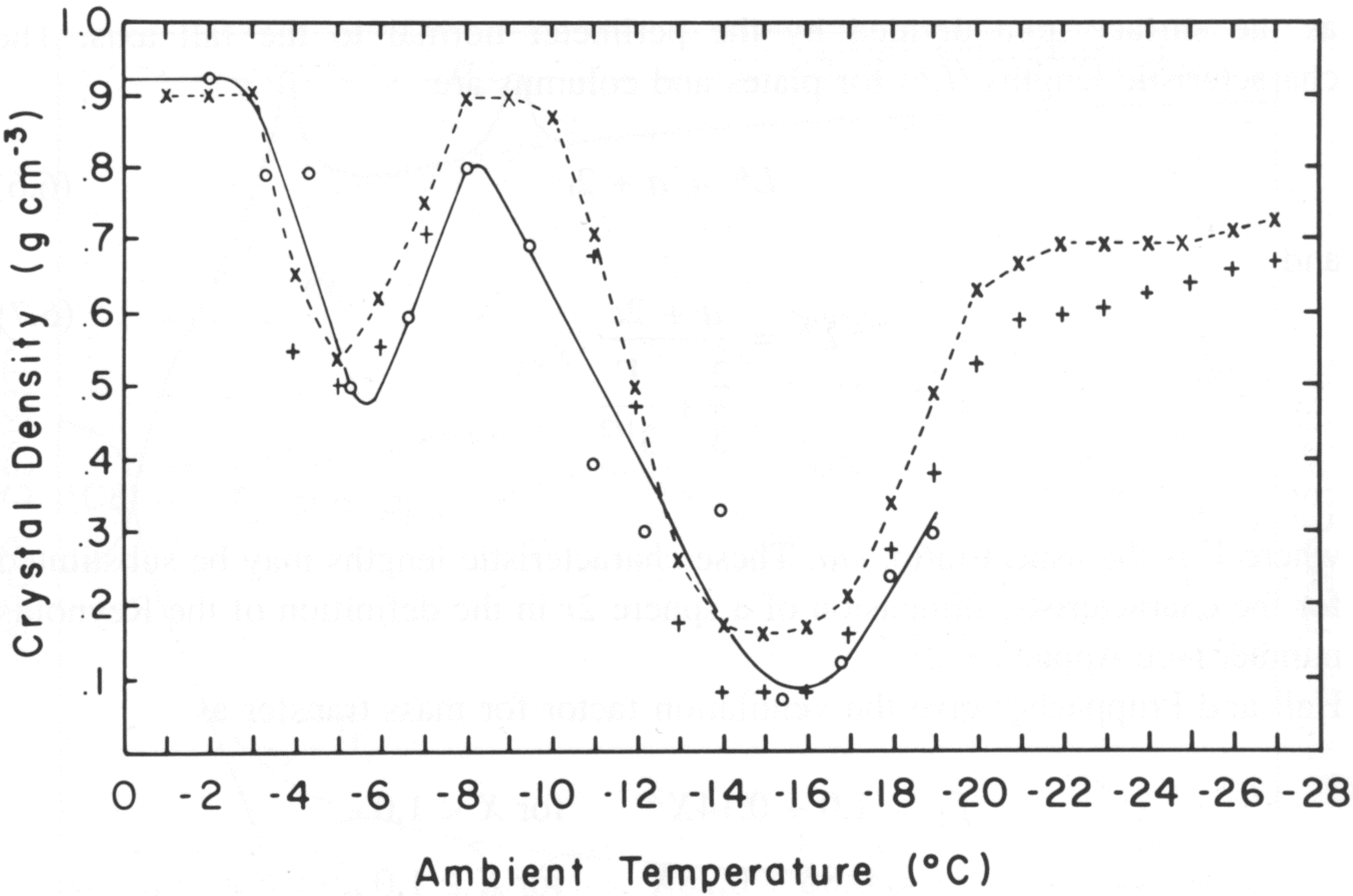
c) Role in rate of mass growth

Ice density tends to decrease as the ice crystal grows.

Table 6.1 Bulk densities for large ice crystals of various habits. (Young 1993)

Crystal habit	**Bulk density ($g\ cm^{-3}$)**
Solid columns	0.9
Hollow columns	0.7
Sheaths	0.6
Needles	0.3
Plates	0.9
Sector plates	0.5
Dendrites	0.16

Ice crystal density as a function of temperature (habit)



Predicted crystal density after 60 s growth at water saturation. Fig. 6.11 of Young (1993)

Growth of ice particles by collection

Collection equation for accretion
dN_s/dt – rate at which water droplets are collected by the ice particle
A – cross sectional area of ice particle
E_{iw} – collection efficiency
n_w – concentration of water droplets
v_i, v_w – terminal fall speeds of ice and water droplets

$$\frac{dN_s}{dt} = AE_{iw}n_w\left|v_i - v_w\right|$$

Fall speeds are complicated, since ice crystals assume many habits (and ice density). The Davies number is often used:

$$N_{Da} = \frac{8mr_c^{\,2}g\rho_{air}}{\eta^2 A}$$

Where m is mass, r_c is the characteristic semi-dimension, A is cross sectional area normal to the fall axis.
There are many forms of Reynolds number that utilize N_{Da}. See Young (1993), pp. 200-202 for more information.

Various degrees of riming of ice particles.

(a) a lightly rimed needle

(b) Densely rimed column

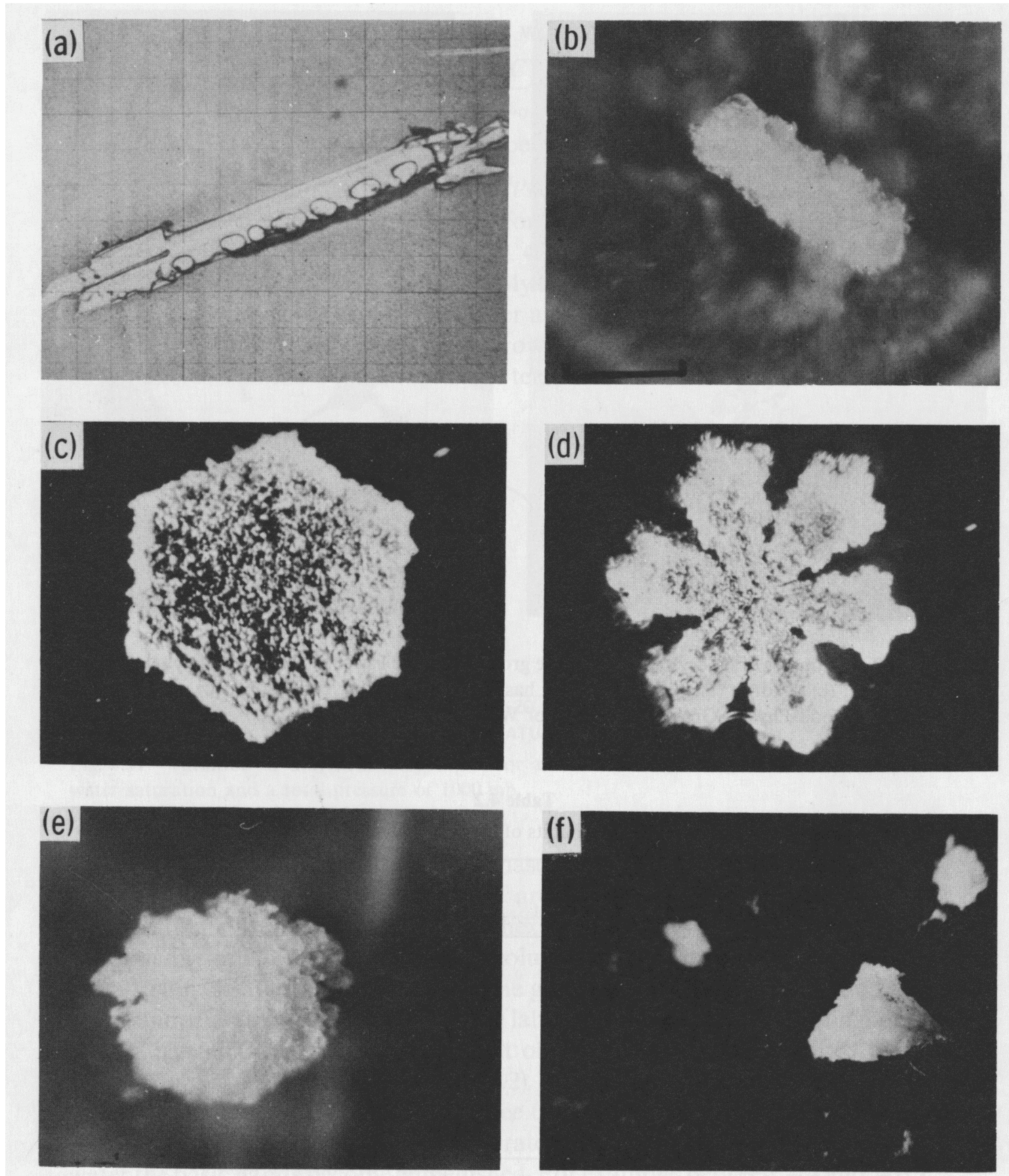
(c) Densely rimed plate

(d) Densely rimed stellar

(e) Lump graupel

(f) Conical graupel

Fig. 4.33 from Wallace and Hobbs 1977



Graupel represents an advanced form of riming. It is a common ice particle in thunderstorms, and often represents the embryo of hail stones.

Characteristic graupel size is several mm (up to ~1 cm).

Example of a conical graupel. Fig. 8.1 from Young 1993, adapted from Nakaya 1954.

Riming

- Riming (accretion) is the counterpart to collection (continuous or stochastic) of cloud droplets by a raindrop.
- When a supercooled cloud droplet is touched by a falling ice particle, the droplet freezes instantaneously and sticks to the ice particle. This occurs because nucleation of ice by ice occurs for $T < 0$ °C.
- Important issues for this process:
  - Terminal fall speed of ice particles
  - Collision / collection efficiencies

Accretion equation (9.8)

- Growth of graupel (hail):
  - m: mass of particle
  - M: cloud water content
  - R: equivalent radius of particle
  - u(R): terminal fall speed
  - E: mean collection efficiency

$$\frac{dm}{dt} = \overline{E} M \pi R^2 u(R)$$

picture

Ice particle terminal fall speeds u(R) are complicated

- Shape factor
- Ice density
- Graupel has a variable density, with a characteristic value of ~0.5 g m^{-3}
- Some ice particles have a fall speed independent of size
- The density correction also applies to V_T of ice particles:
  - $(\rho_0/\rho)^{0.4}$

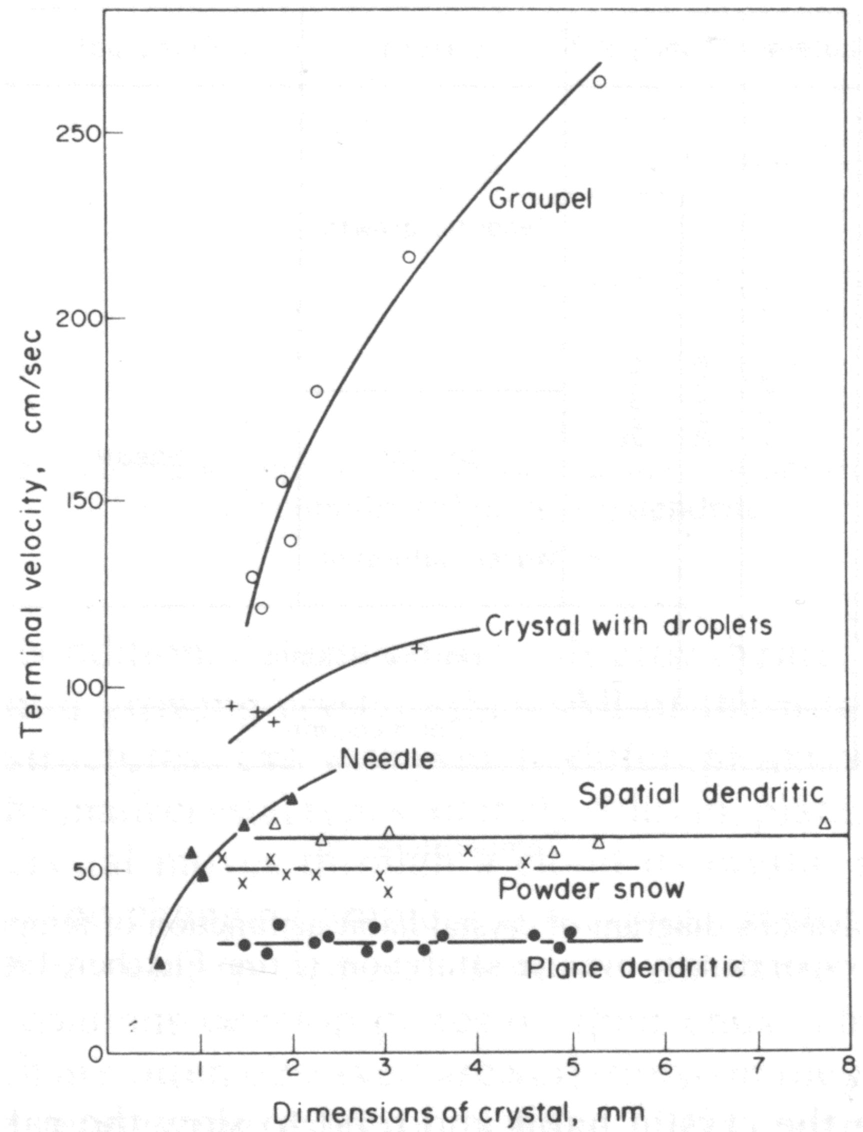
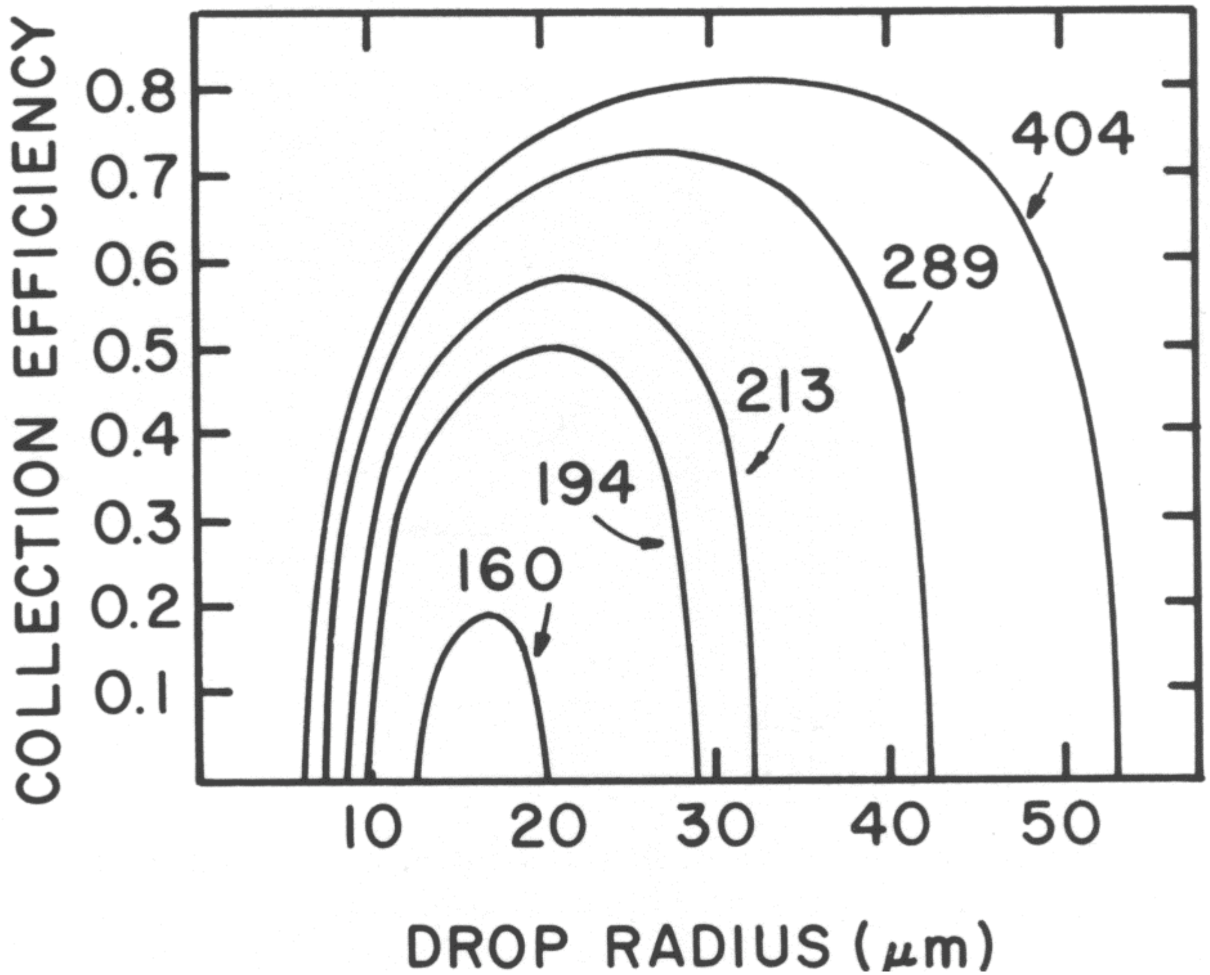


Fig. 9.7 from Rogers and Yau.

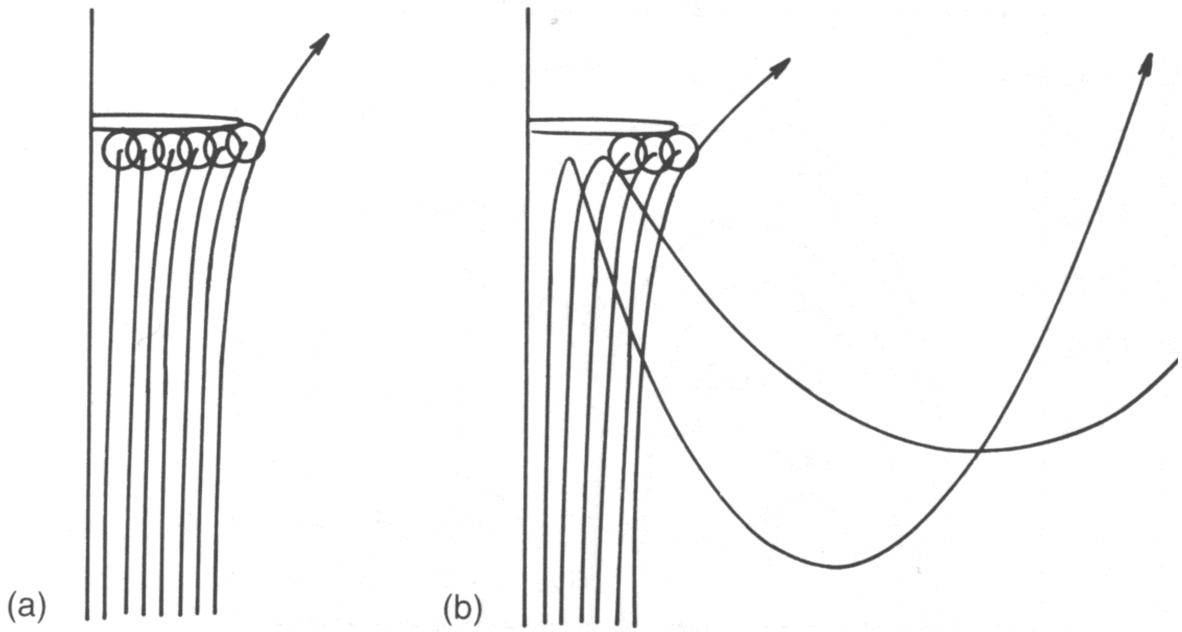
Collection Efficiency

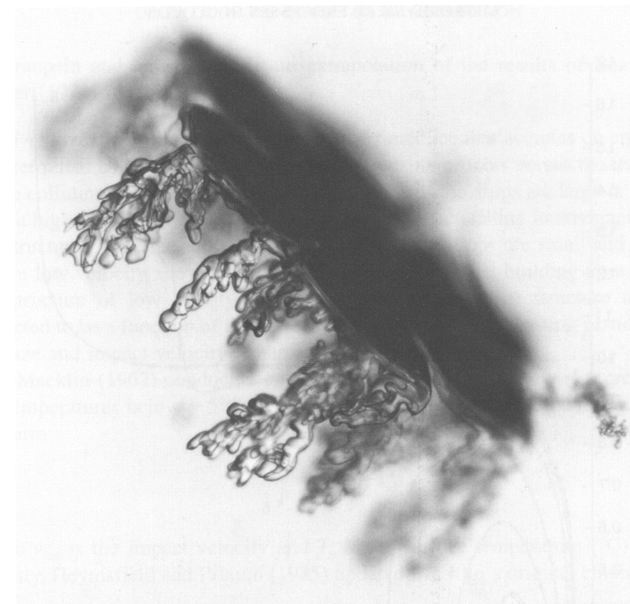
Small droplets will tend to evaporate and follow airflow streamlines around the falling ice particle. Droplets < 1 μm in size are not collected.

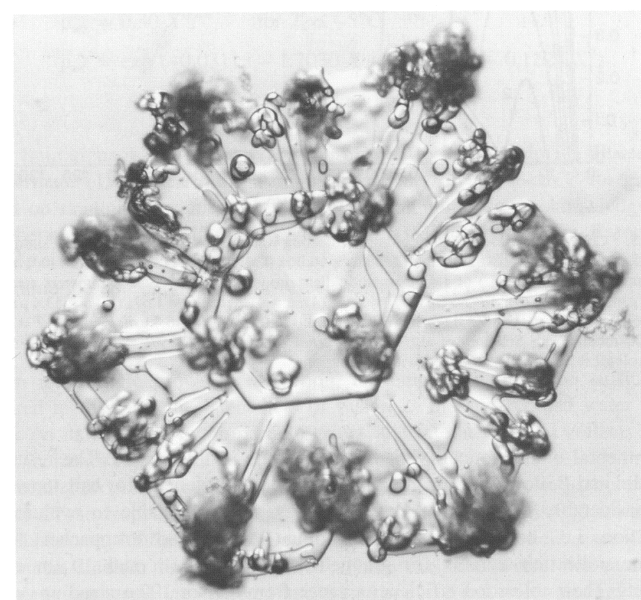
Collection efficiencies for plates do not become large until a diameter of 150-200 μm is attained. See Fig. on the right.



Collection efficiency for thin plates collecting water droplets, based on numerical simulations by Pitter and Prupacher (1974), as a function of drop radius. Plate sizes are labeled in μm. Fig. 8.2 from Young 1993.







Above: Trajectories of droplets interacting with the relative airflow of a falling plate (a = 404 μm) as described by numerical simulations of Pitter and Pruppacher 1974. In (a), the droplets have radius of 52 μm. In (b), the droplets have radius of 52.5 μm. Fig. 8.3 from Young 1993.

Right: Rime deposits on a plate crystal. (a) side view. (b) bottom view (the side on which the droplets were captured). Fig. 8.4 from Young.

Collection efficiencies for columns collecting water droplets, based on numerical simulation, as a function of drop radius. The smallest column (1) of radius (a) 23.5 μm and aspect ratio (Γ) of 1.43 does not collect a drop. Size parameters are:

(2) 32.7 μm, Γ = 1.43

(3) 36.6 μm, Γ = 1.54

(4) 41.5 μm, Γ = 1.67

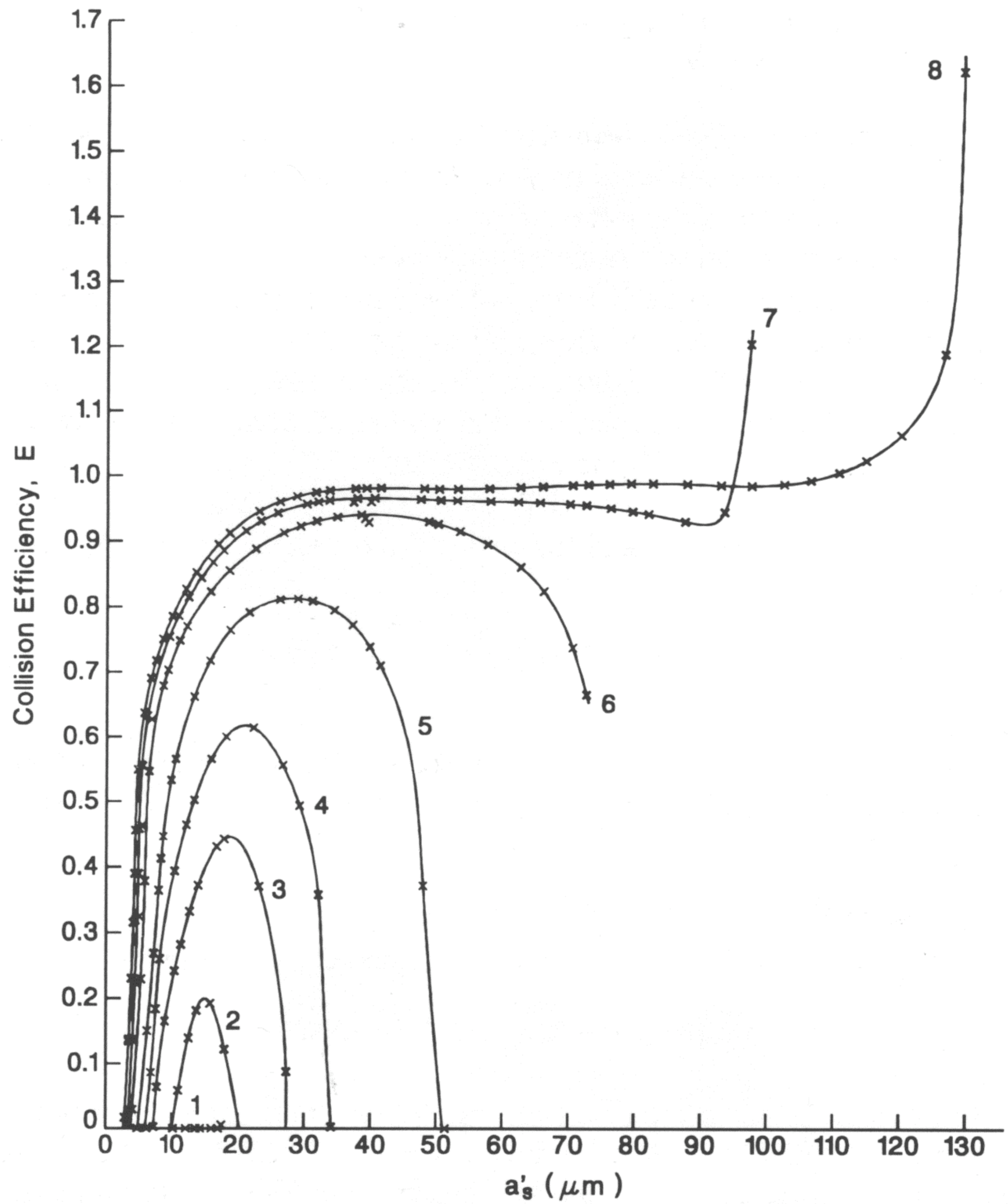
(5) 53.4, 2.22

(6) 77.2, 3.33

(7) 106.7, 5.0

(8) 146.4 μm, Γ = 8.33

Fig. 8.5 from Young 1993.



Growth of hail

Continuous growth equation applies here.

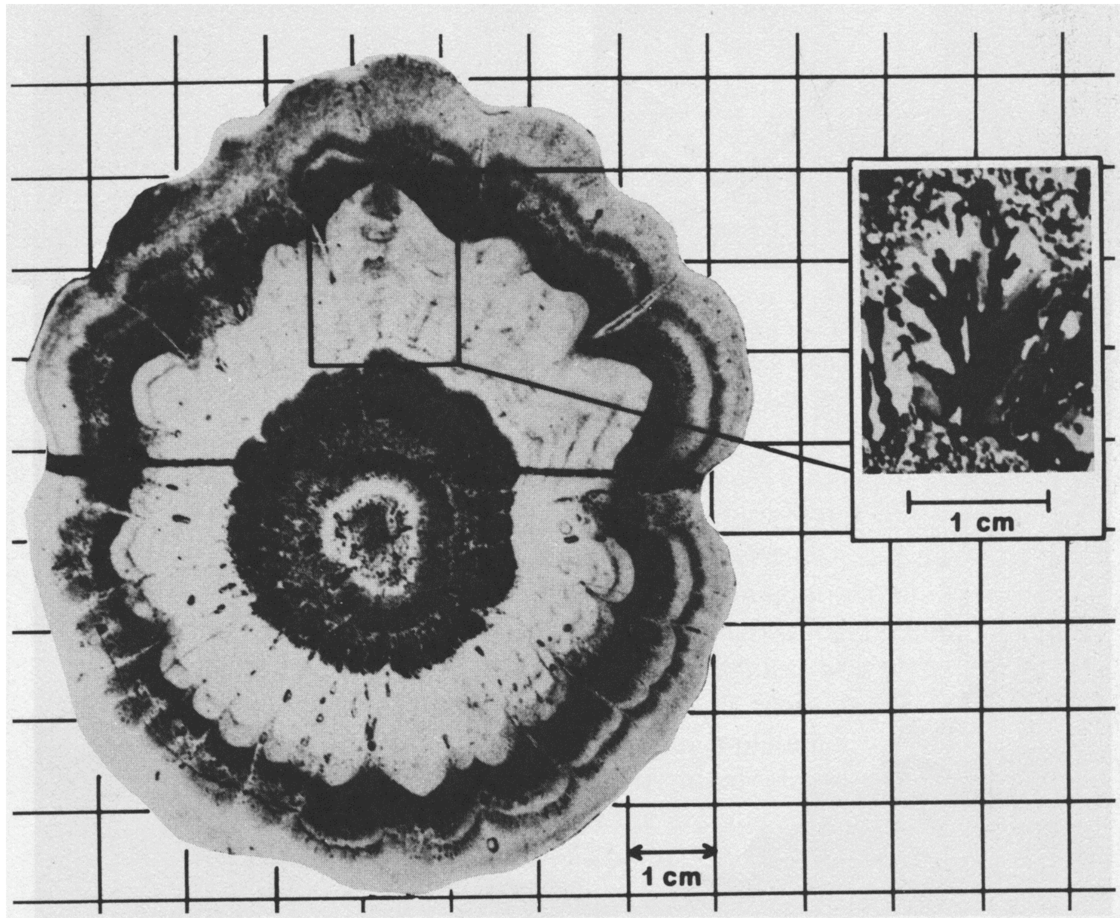
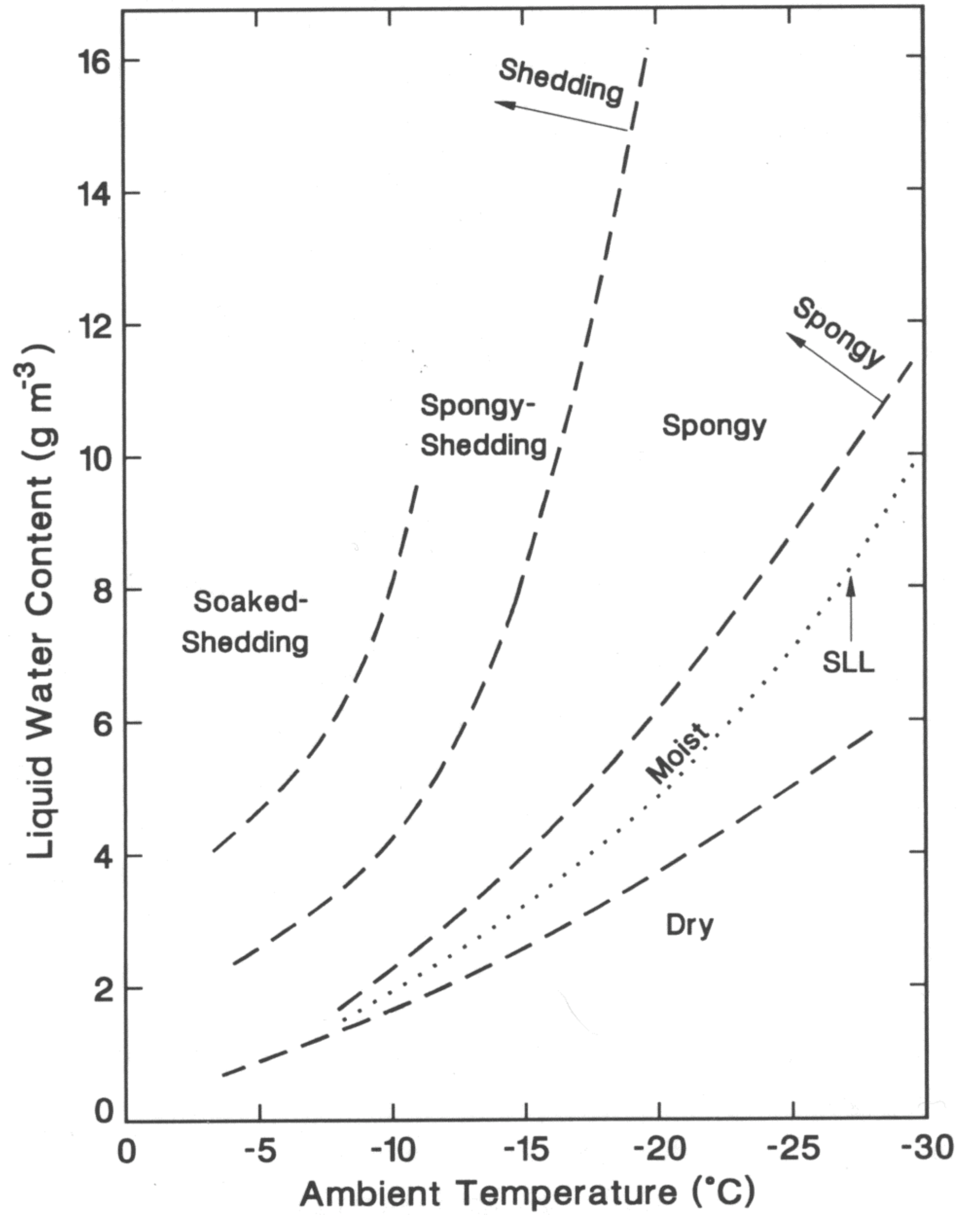


Fig. 4.34 Thin section through the growth center of a natural hailstone. [From *Quart. J. Roy.*

Five observed hailstone growth regimes as a function of liquid water content and temperature. The dotted line represents the Schumann-Ludlam limit. Fig. 8.7 from Young 1993.



Growth of snow flakes: aggregation

Aggregation: the collision and sticking of two or more ice crystals.

Dependence on:

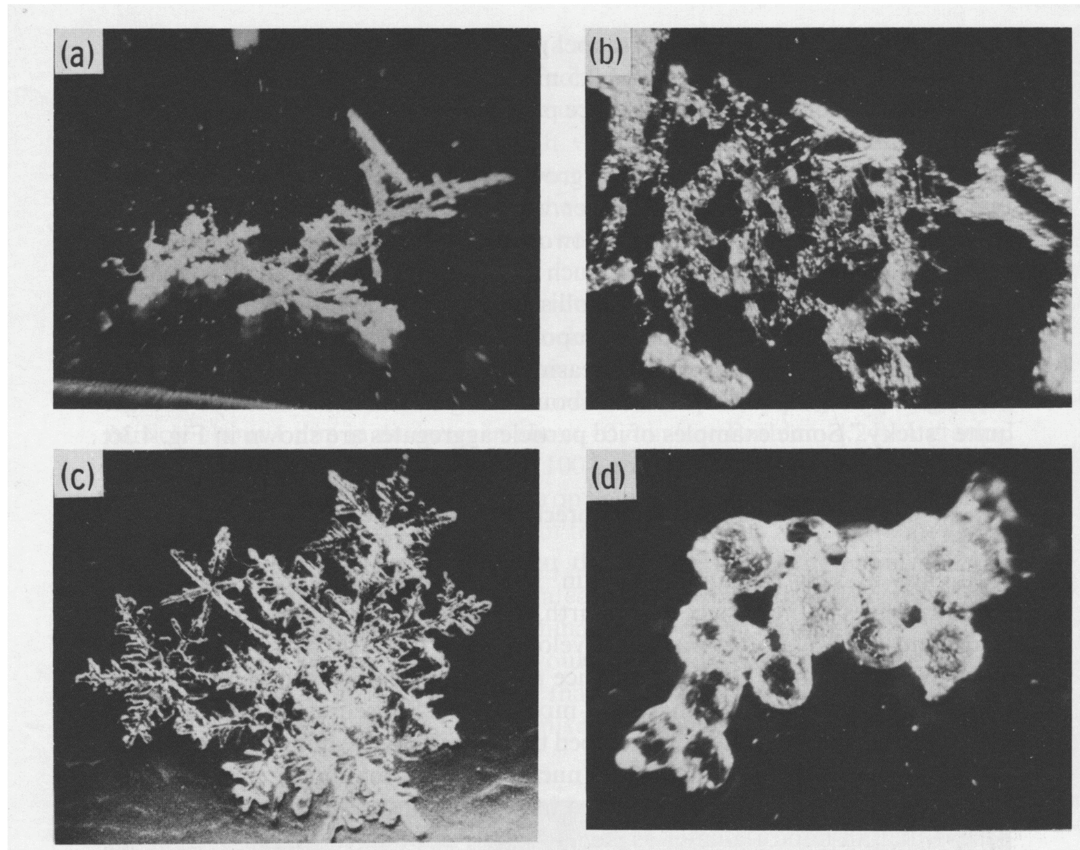
a) Temperature

b) Crystal habit (dendrites are ideal)

Physics:

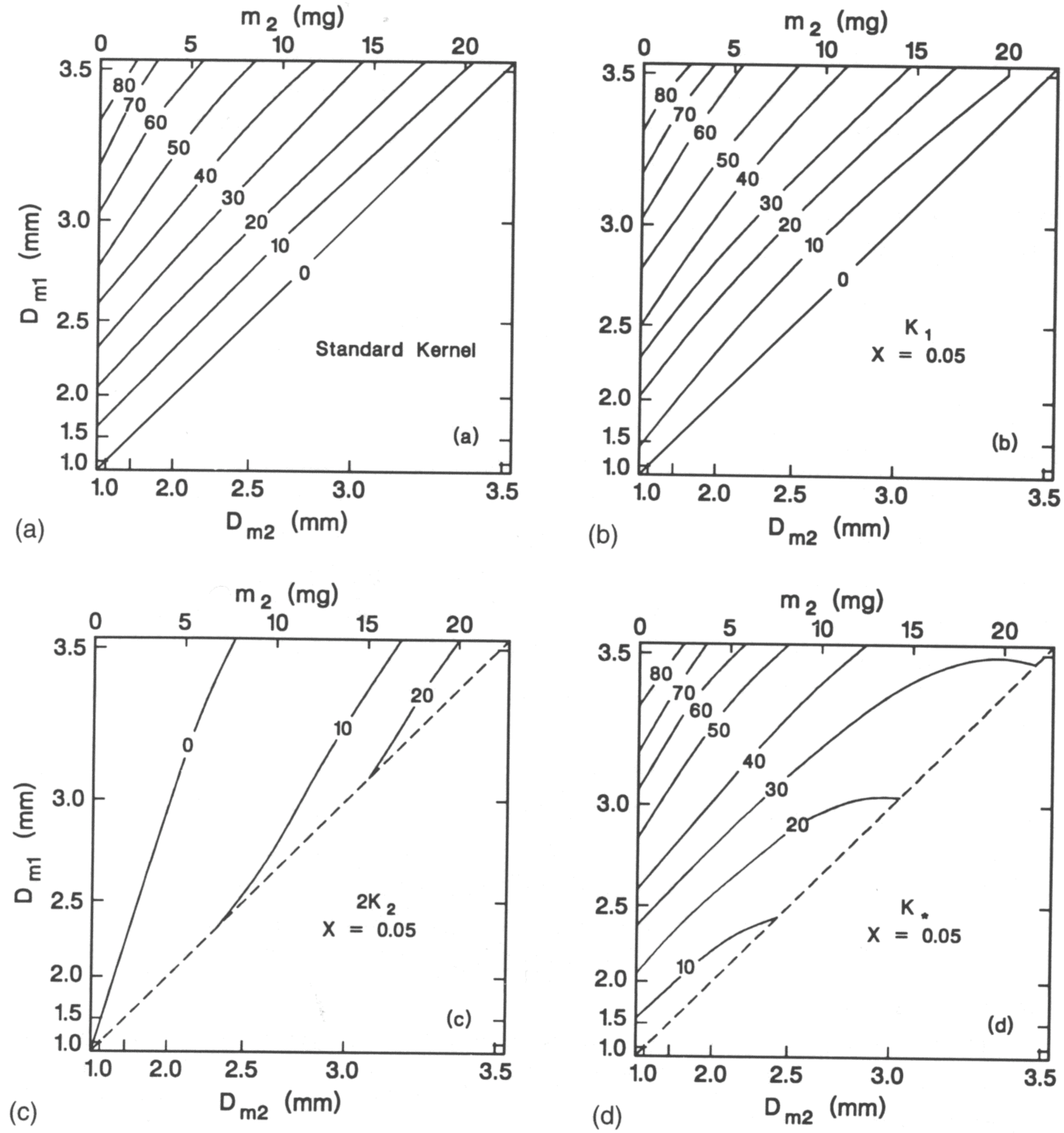
a) Collisions
b) Sticking probability

$$\frac{dm}{dt} = EM\pi R^2 \Delta u$$



Examples of aggregates: (a) densely rimed needles, (b) rimed columns, (c) dendrites, and (d) rimed raindrops. Fig. 4.36 from Wallace and Hobbs 1977.

Modified collection kernel for aggregation is partitioned into

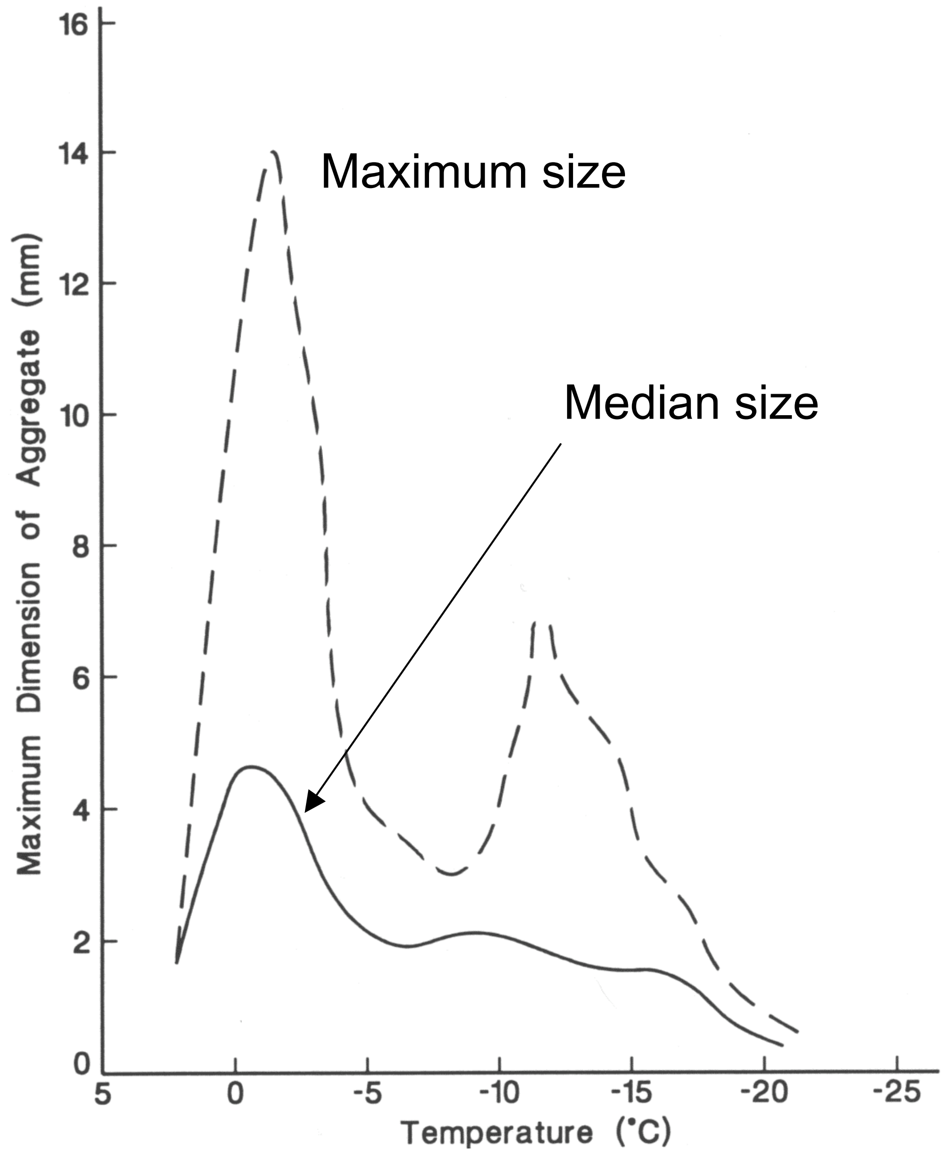


Maximum dimensions of aggregates as a function of T based on aircrafts observations of Hobbs et al (1974). Solid curve: median, dashed line: largest aggregates. Fig. 8.9 from Young 1993.

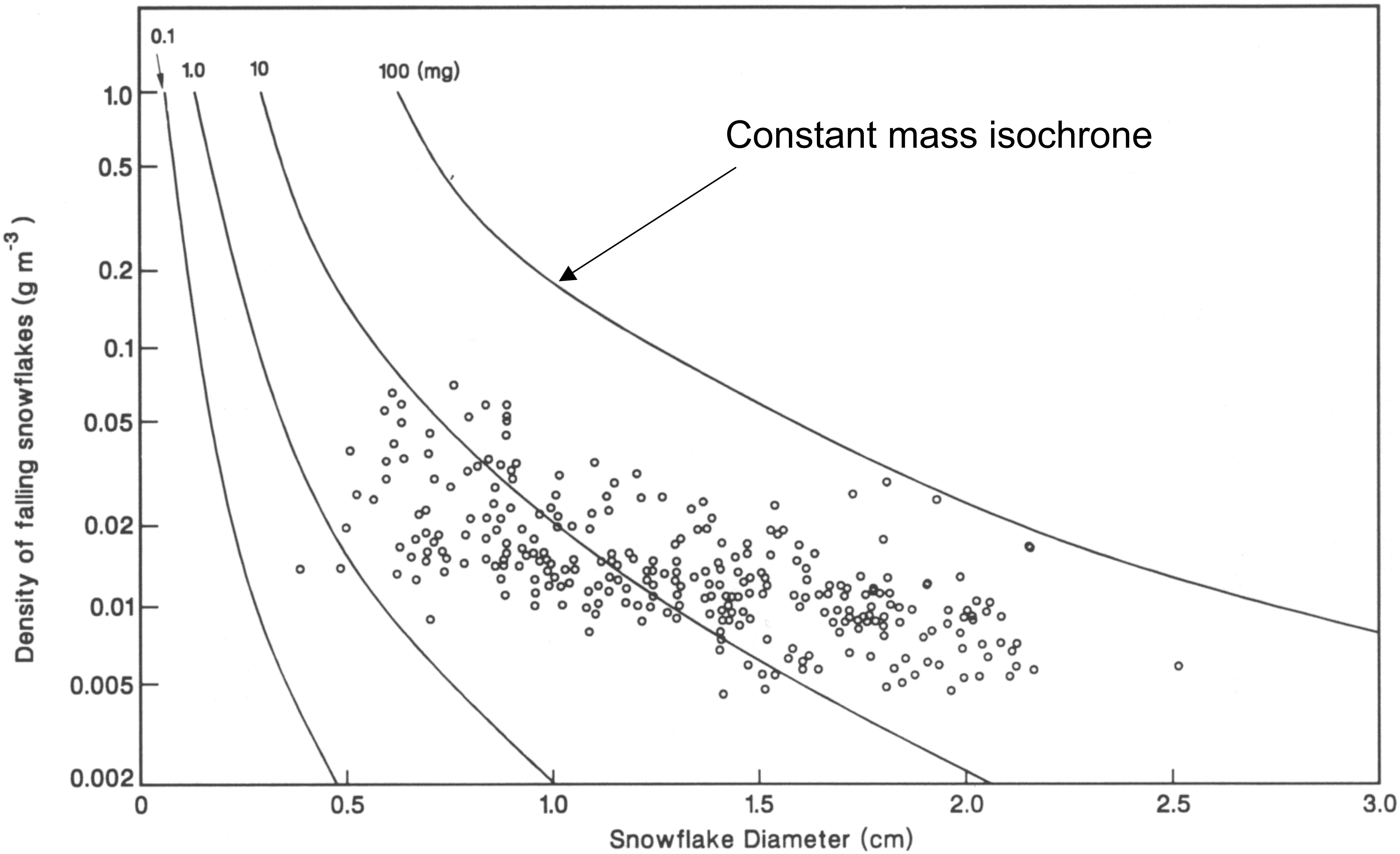
Aggregation is a function of:

a) temperature, which influences the "stickiness" of ice. Maximum between 0 and -5 °C.

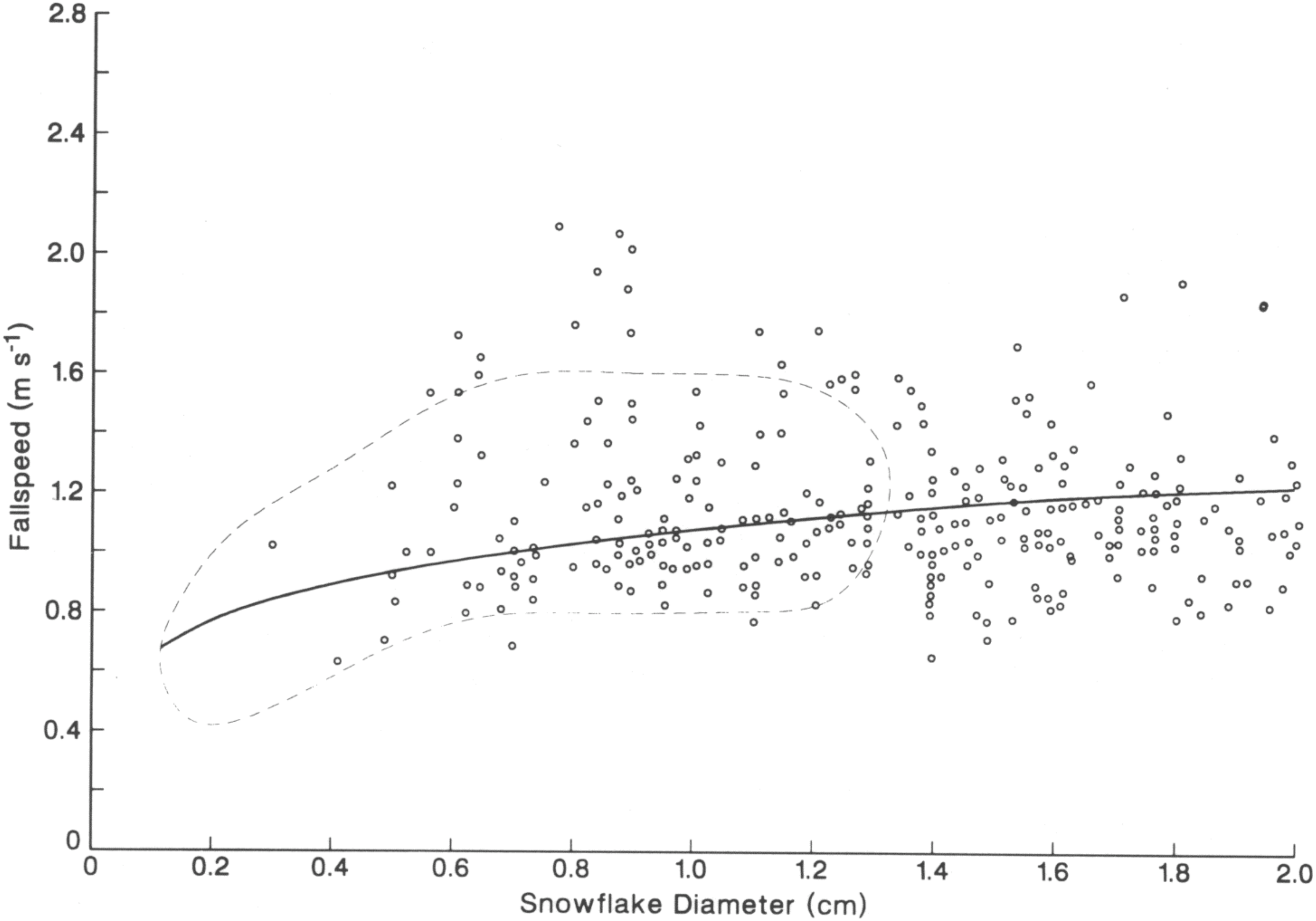
b) Crystal habit (implicitly temperature): Dendrites aggregate more effectively because their branches can become intertwined.



Snowflake density (dendrite aggregates) as a function of diameter



Snow flake fall speed vs. diameter



Melting of ice, figs 8.13, 8.14

Ice particles in the melting region shown in Fig. 8.13 may be expected to accumulate liquid water. If the ice particle were solid initially, liquid water would be expected to accumulate on its surface. Thus, continued melting of the ice particle requires that sensible heat be conducted to the liquid surface of the particle and then through the water shell to the ice interior. Mason (1956) assumes that the liquid coating is symmetrically distributed over the surface of the ice particle. The melting rate given by Mason is

$$
\begin{aligned}
L_f \left.\frac{dm}{dt}\right|_{\text{melt}} &= \frac{4\pi r_w r_i K_w T_x}{r_w - r_i} \\
&= 4\pi r_w \left[K f_2^* (T_\infty - T_x) + L_v f_2 (\rho_\infty - \rho_{s,x}) \right],
\end{aligned}
\tag{8.11}
$$

where r_w and r_i represent the radii of the outer water shell and the inner ice core, respectively, and K_w is the conductivity of liquid water. Note that the conduction to the melting particle is based on a surface temperature (T_x) that is above 0°C.

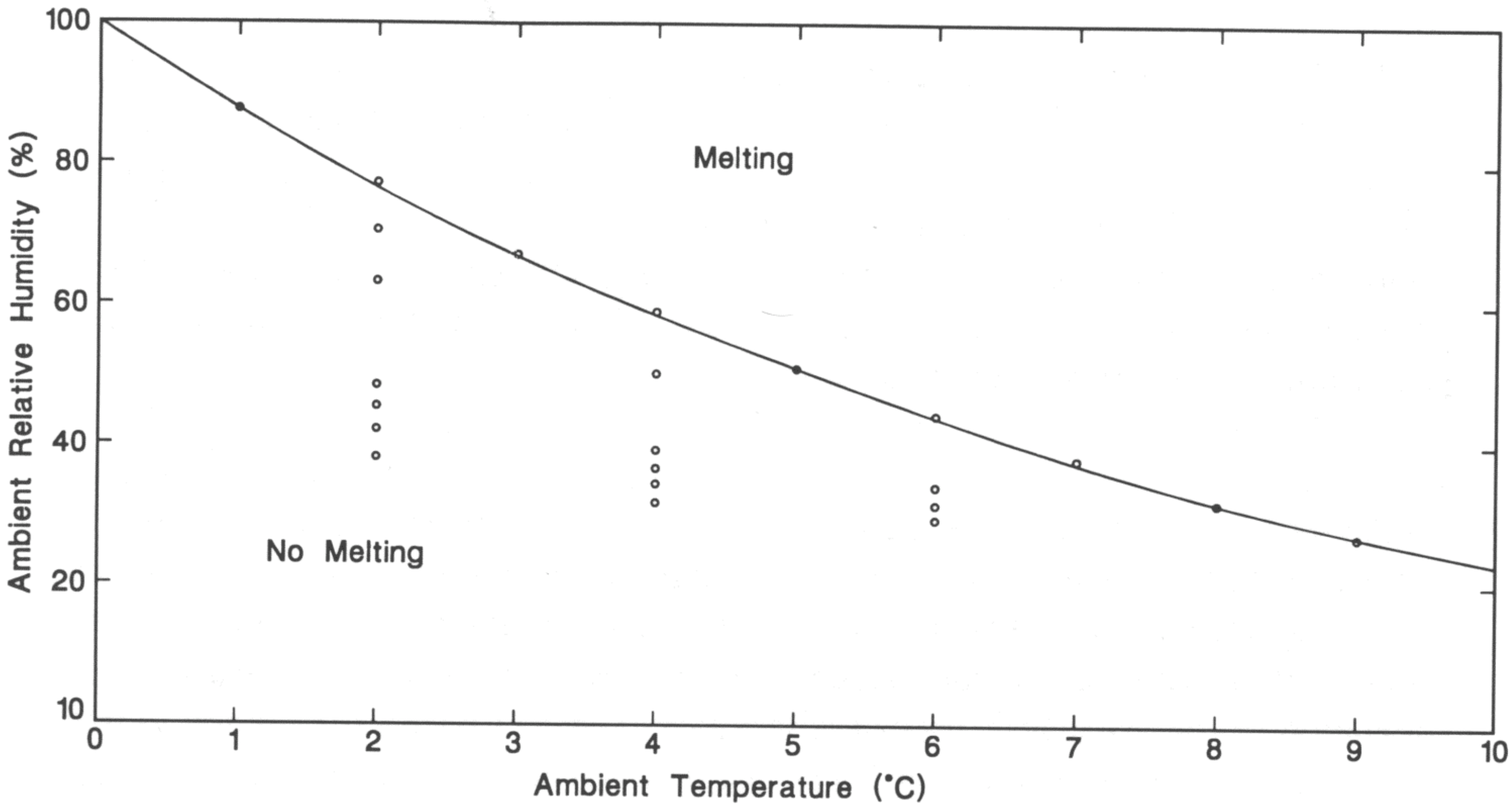


Fig. 8.13. The critical conditions for melting as a function of ambient temperature and relative humidity at 700 mbar as calculated based on (8.11). Points represent

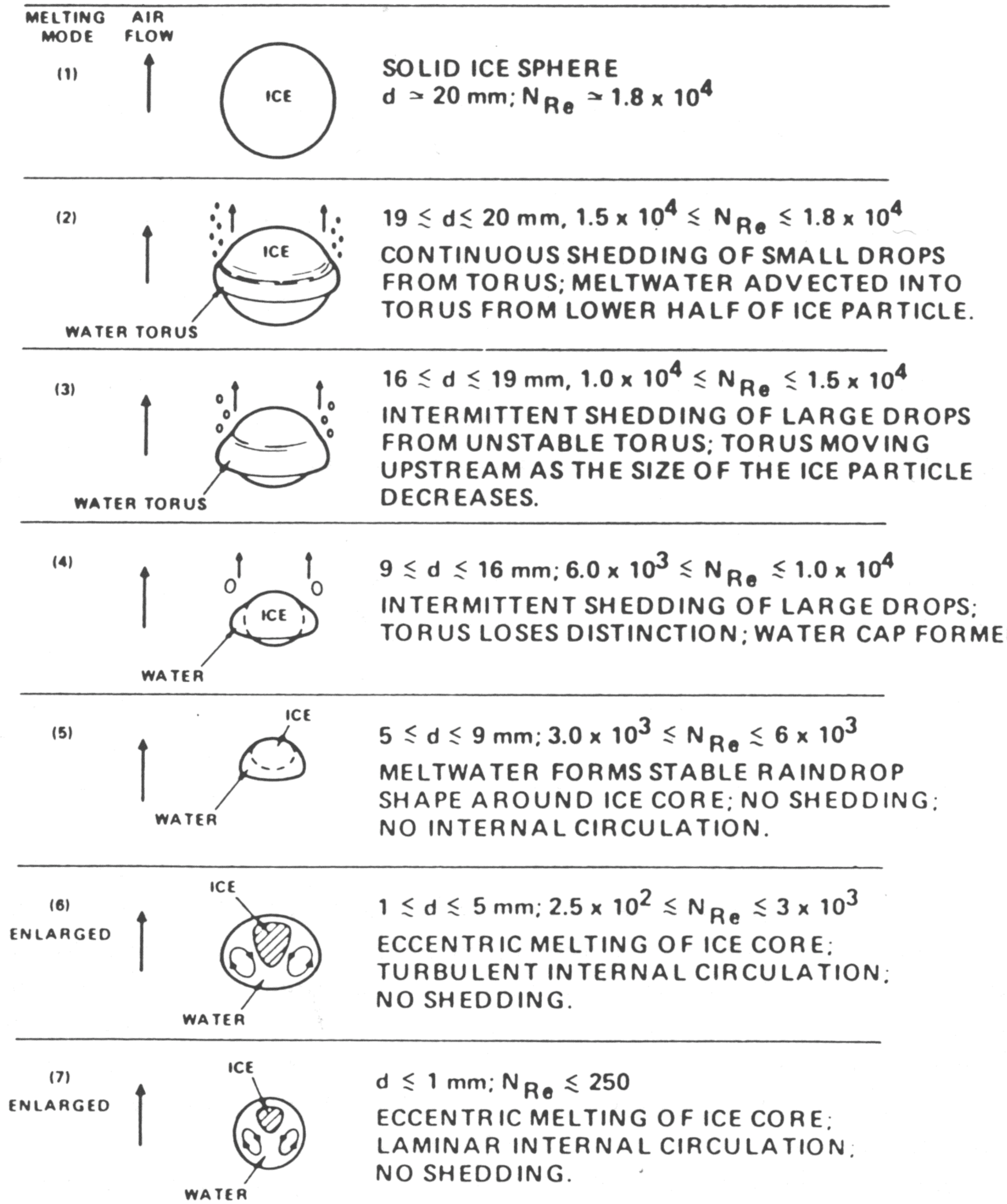


FIG. 8.14. Schematic of the melting modes. From Rasmussen et al. (1984b). *Journal of Atmospher Sciences*, **41**, 3. American Meteorological Society. Reproduced with permission

Snowflake size distribution

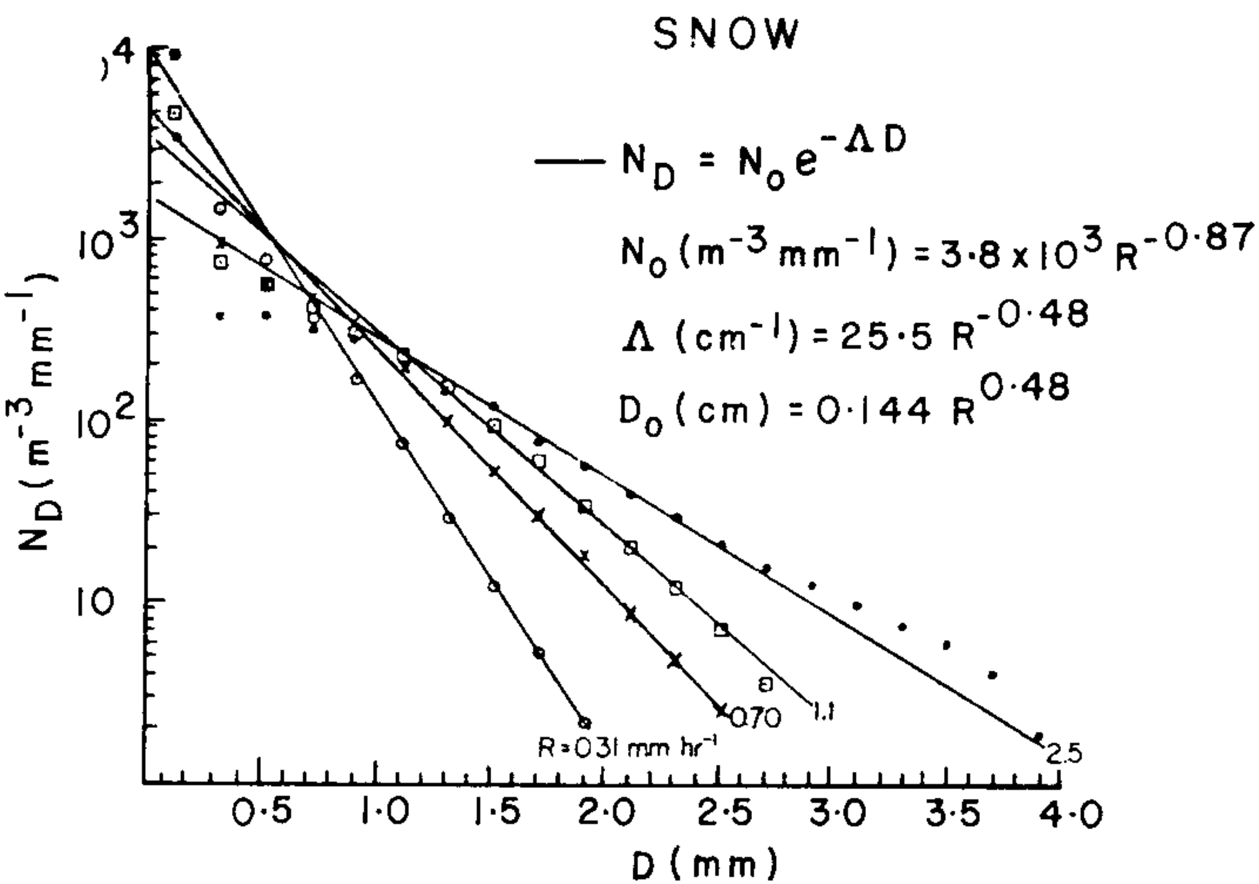
R&Y, pp. 180-183

Exponential size distribution, as for rain, works reasonably well:

$N(D) = N_0 e^{-\Lambda D}$

$\Lambda(\text{cm}^{-1}) = 25.5R^{-0.48}$

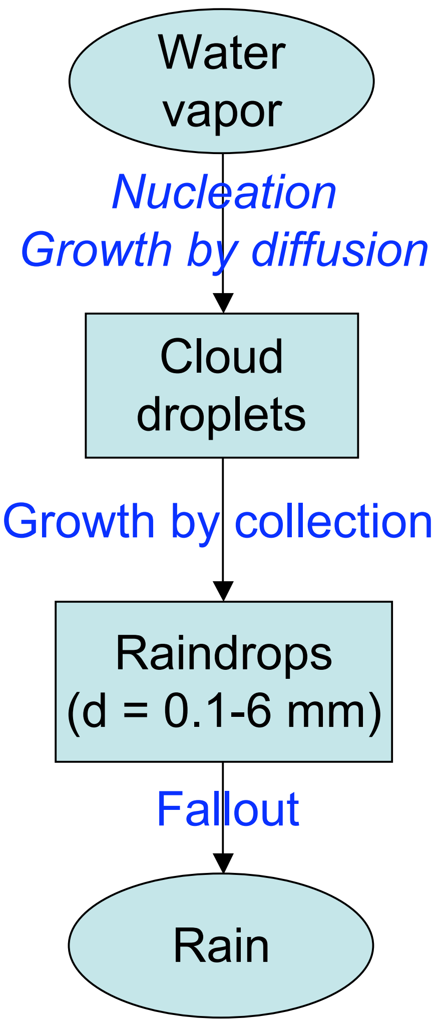
$N_0\ (\text{cm}^{-4}) = 3.8 \times 10^{-2}\ R^{-0.87}$



A review of where we are on this topic

Representation of "paths to precipitation"

We considered in Chaps. 7-8:



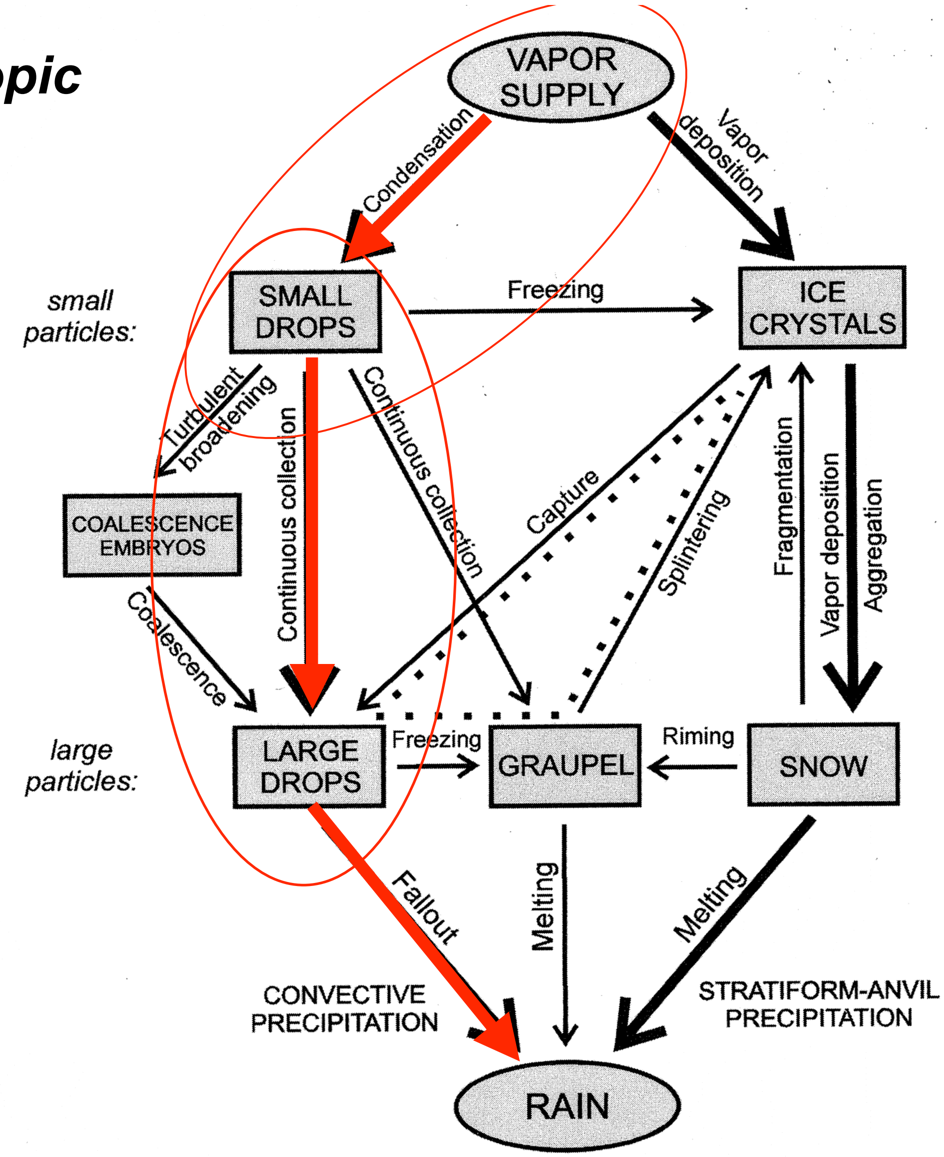


FIG. 8.10. A simplified schematic diagram of possible interactions between various categories of cloud particles. The bold arrows show the dominant pathways by which water vapor is transformed into rain via the "warm-rain" process (left-hand side) and via the "cold-rain" process (right-hand side) in heavy rain situations. The dotted set of arrows near the center of the diagram identifies a likely cyclical process for generating secondary ice particles via the rime-splintering mechanism of Hallett and Mossop (1974). The form of the diagram was adapted from Rutledge and Hobbs (1984).

Homework, RY Chap. 9:

1. 9.1

2. Sketch 3 possible paths to rain formation involving ice-phase microphysics. For each branch of the path, write the relevant equation.

3. Examine the material at the web site
http://www.its.caltech.edu/~atomic/snowcrystals/class/class.htm